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RESEARCH MEMORANDUM

GENERAL TREATMENT OF COMPRESSIBLE FLOW IN EJECTORS AND EXAMPLE OF ITS APPLICATION TO PROBLEM OF EFFECT OF EJECTOR ADDITION ON THRUST OF JET-PROPULSION UNITS

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SUMMARY

The principle object of the present work is to present a general treatment of compressible flow in ejectors with friction in the mixing tube included. A secondary object is to show the application of the flow theory to the calculation of the effect of ejector addition on thrust of jet-propelled airplanes and missiles. The calculations made did not explore the ejector possibilities completely but covered a few conditions that were considered appropriate to modern turbojet and ram-jet airplanes and missiles. Applications to some other problems are also briefly discussed.

The general treatment of compressible flow in ejectors led to a theory with certain limitations which permits calculation of physical conditions in ejectors for varying geometries for any set of flow actuating conditions. Certain assumptions, for example, the assumption of perfect mixing of the fluids in the ejector are of questionable validity. For practical purposes, however, it is thought that satisfactory agreement between the theoretical conditions and experimental results will be obtained. The general theory can be applied to any problem concerned with pumping or thrust augmentation to be obtained through use of ejectors.

The exploration of the advantage to be gained through use of ejectors, whose geometry was varied to obtain ideal conditions through the flight range, to augment the thrust of jet-propulsion units for the limited range of conditions used in the calculations, showed that generally at take-off but otherwise only in isolated instances for certain combinations of flight and engine conditions was the thrust increased. It was determined that when an ejector with a nozzle at the mixing-tube exit was added to a ram-jet installation operating at supersonic flight speeds for the direct purpose of cooling or boundary-layer control, good thrust augmentation was obtained as an indirect result. For the direct purpose of thrust augmentation,

however, more thrust was obtained with a large ram-jet installation without an ejector whose mass flow rate was equal to the sum of the primary and secondary flow rates in the ejector setup. In the calculations it was necessary to rule out certain solutions that were considered physically impossible and for which very optimistic thrust augmentation results were obtained. Previously reported optimistic calculated results of other investigations very probably are based on such physically impossible solutions.

INTRODUCTION

The ejector is an aerodynamic device that has offered possibilities of improvement of the performance of airplanes by its application. Much interest has been shown by aircraft designers in recent years in the use of ejectors fitted to the exhaust stacks of reciprocating engines and in the tail pipes of jet engines for thrust and cooling augmentation. The principle involved in the case of thrust augmentation is as follows: Thrust is equal to mass of fluid flowing times the change of velocity of the fluid from the free stream to a position where the fluid issuing from the jet has again a static pressure equal to that of the free stream. The kinetic energy wasted in the wake diminishes as the mass flow engaged increases, with consequent increase in propulsive efficiency. When the gain in propulsive efficiency through engaging additional air by ejector action exceeds the loss of efficiency in the ejector itself, thrust is augmented. The principle of the ejector as a cooling augmentation device is that transfer of momentum from a high-velocity exhaust-gas jet to low-velocity air creates an action on the low-velocity air such that it is pumped from a low-pressure to a high-pressure region at the ejector exit. The low-pressure region is the space behind an engine, the reduction of pressure in the space causing more cooling air to flow across the engine than if the ejector is not used. An example of an ejector placed at the exit of a nozzle downstream of an engine for thrust-augmentation purposes is illustrated in figure 1 for a typical jet-propelled airplane.

Many investigations, both analytical and experimental, have been made for the purpose of determining the performance of ejectors. Most of the investigations have been made for the purpose of determining the pumping effect of ejectors, actuated by internal combustion engine exhaust gases, on the cooling air. The complexity of the problem is such, however, that assumptions made regarding friction and compressibility have limited in one way or another the applicability of the various theories evolved. The underlying principles of thrust augmentation were set forth by Schubauer as early as 1933 (reference 1), together with the results of low-speed wind-tunnel tests of small-scale jets. Schubauer observed that the thrust augmentation diminished with increasing flight speed, and concluded from his tests that the augmentation was insufficient to permit jets to compete with propellers at the flight speeds of that period. Flügel (reference 2) developed a theory for the design of jet pumps which checked well with his experimental results in the range explored. Jacobs and Shoemaker (reference 3), from static thrust measurements of a Melot type thrust augmentor, arrived at substantially the same conclusions as Schubauer. Lee (reference 4) presented a theory due to Keenan and pointed out that static thrust increases as high as 80 percent were theoretically possible. The same reference reports a 35-percent augmentation of exhaust-stack thrust obtained with ejectors fitted to a Pratt & Whitney R-1830 engine. Morrisson (reference 5) also in model tests noted large augmentation of static thrust when the induced air stream reached the ejector at about atmospheric pressure and with negligible velocity. When the approach pressure or velocity of the induced air stream increased, the augmentation of thrust fell off rapidly. In Great Britain, A. R. Howell, B. H. Slatter, and W. Bailey obtained similar findings substantiated by a limited number of tests, based on a more comprehensive theory than those of others. Their theory indicated that tests with unheated actuating jets would not properly simulate conditions with a heated jet, and experiments demonstrated that static thrust augmentation diminished as the temperature of the actuating jet was elevated. Reappearance of substantial thrust augmentation at very high flight speeds was another outcome of this analysis, the theory showing augmentation as high as 400 percent. Nevertheless, the authors did not stress this point, and concluded that practical ejector thrust augmentation could be obtained only at very low speeds.

For flight in the transonic and supersonic speed range, where the more comprehensive theories suggested but for which experiments are lacking to confirm the possibility of substantial thrust augmentation, the simpler analyses were inapplicable. The solution of this problem, together with that of the design of ejector pumps for the same high flight speeds, requires an analysis of the aerodynamics of the ejector, sufficiently comprehensive to apply without limitations on speed of flight or internal air flow. The present report contains such an analysis, together with a study of thrust augmentation sufficient to demonstrate its application and indicate some of the more important trends. The method of application of the theory of flow in the ejector to the problem of cooling augmentation for a continuous jet is also briefly discussed as another application of the analysis.

In the development of the theory, the assumption of complete mixing of the air and the gas in the ejector mixing tube was made.

The assumption of no losses in the duct and nozzle systems feeding the primary and secondary air to the ejector was also made in order to determine whether under ideal conditions thrust augmentation would be obtained and to dismiss the system losses from the theory because it is another problem. Friction in the ejector, however, was considered. Calculations were made, using the theory, of the thrust increase at a few conditions that are considered appropriate to turbojet airplanes and ram-jet airplanes and missiles. The range of conditions chosen was considered sufficient to demonstrate the application of the theory to this particular problem. The results of the calculations are presented in curves given in the report.

SYMBOLS

A sketch of the jet system illustrating the symbols used herein is given in figure 2. The supersonic nozzle at the mixing-tube exit is shown with broken lines to symbolize the fact that the ejector may be used with or without the nozzle.

A cross-sectional area, square feet

a speed of sound in air, feet per second $\left(\sqrt{\frac{\gamma p}{\rho}} = 49\sqrt{T}\right)$

c_p specific heat of air at constant pressure, Btu per pound per °F

c_v specific heat of air at constant volume, Btu per pound per °F

d diameter of mixing tube, feet

f friction factor $\left(f = 0.049/(Re)^{0.2}\right)$

g ratio of absolute to gravitational unit of mass, pounds per slug (32.17)

H absolute total pressure of fluid, pounds per square foot, absolute

J mechanical equivalent of heat, foot-pounds/Btu, (778)

l length of mixing tube, feet

M Mach number (V/a)

m mass rate of flow of fluid, slugs per second

p absolute static pressure of fluid, pounds per square foot, absolute

q dynamic pressure of fluid, pounds per square foot

R gas constant for air, foot-pounds per $^{\circ}F$ per pound (53.33)

r pressure-rise ratio from free stream to jet-unit exit $\left(H_u/H_o\right)$ (for 100 percent efficiency of diffuser at jet-unit inlet, r equals pressure-rise ratio across jet unit)

T absolute temperature of fluid, $^{\circ}F$ absolute

V velocity of fluid, feet per second

X ratio of secondary-air flow area at mixing-tube inlet to area at jet-nozzle exit $\left(A_a/A_j\right)$ $\left(A_a + A_j = \text{area of mixing-tube cross section}\right)$

γ ratio of specific heats of air $\left(c_p/c_v\right)$, (1.4)

θ ratio of mass of air flowing to mass of gas flowing with ejector $\left(m_a/m_j\right)$

ρ density of fluid, slugs per cubic foot

ϕ ratio of thrust with ejector to thrust without ejector when mass of gas m_j flowing is same in both cases and ejector exit pressure equals pressure of atmosphere

ϕ' ratio of thrust with ejector to thrust without ejector when mass of gas m_j flowing is same in both cases but ejector exit pressure does not equal pressure of atmosphere

Δp_f static-pressure loss in mixing tube caused by friction, pounds per square foot

ΔT temperature rise in combustion chamber of ram jet, $^{\circ}F$

Δ denotes increment

Re Reynolds number in mixing tube

Subscripts:

a refers to conditions of air at mixing-tube inlet

b refers to back pressure conditions in tank at ejector exit

d refers to conditions at discharge of nozzle placed downstream of mixing tube

e refers to conditions at exit of straight circular mixing tube

j refers to conditions at jet-nozzle exit with ejector

JF refers to conditions at jet-nozzle exit without ejector (free jet)

n refers to conditions at jet-nozzle minimum section when supersonic flow occurs in nozzle

o refers to free-stream conditions

s refers to stagnation temperature

t refers to conditions at mixing-tube nozzle minimum section when supersonic flow occurs in nozzle

u refers to conditions at exit of jet unit (upstream of jet-nozzle exit)

GENERAL TREATMENT OF COMPRESSIBLE FLOW IN EJECTOR

A diagrammatic sketch of a typical ejector is shown in figure 2 from which it is possible to obtain a brief conception of the flow processes involved in an ejector. In the example shown, primary augmentation gas is flowing through the central nozzle and secondary air is flowing through the annular space around this nozzle. The two flows come together at the mixing-tube inlet and mix in the tube shown, the primary air giving up energy to the secondary air and causing the latter to flow through the system. The nozzles and the mixing tube are circular and the latter is also cylindrical. The air and gas mixture either discharges at the mixing-tube exit or flows through a circular nozzle at the mixing-tube exit, as shown in figure 2, and then discharges. The development of the basic relations involved in the flow through such a system will be the purpose of the analysis of this report.

Basic Relations

Assumptions.- Several assumptions are made in the development of the analysis as follows:

1. Reversible adiabatic expansion from jet-unit exit to jet-nozzle exit, from free-air stream to secondary-air nozzle exit (hereinafter called mixing-tube inlet nozzle), and from mixing-tube exit to mixing-tube exit nozzle discharge; or symbolically $H_j = H_u$, $T_{js} = T_{us}$, $H_a = H_o$, $T_{as} = T_{os}$, $H_d = H_e$, $T_{ds} = T_{es}$.

2. Uniform velocity and pressure at inlet of mixing-tube exit nozzle and exits of inlet nozzles. (If no nozzle is used at mixing-tube exit, assumption still applies to mixing-tube exit.)

3. Complete mixing of air and gas before reaching the mixing-tube exit and no heat transfer across the boundary of the mixing tube.

4. Air properties are applicable to gas and to gas and air mixtures. Values of c_p and γ used are constant and equal to 0.24 and 1.4, respectively.

Assumption 1 postulates frictionless, shockless flow which practically can never be obtained. The assumption was made to simplify the problem and to determine the thrust augmentation for ideal conditions. In general, at supersonic flight speeds it would be expected that it would be difficult to design low loss ducts and nozzles in which the velocity is required to diminish appreciably.

Conditions in mixing tube.- Three fundamental equations - the conservation of energy, the conversation of momentum, and the conservation of mass equations - are used to determine the conditions at the mixing-tube exit in terms of conditions at the mixing-tube inlet. Applying the law of conservation of energy and with assumptions 2, 3, and 4, the following equation can be set up:

$$m_a\left(Jgc_pT_a + \frac{V_a^2}{2}\right) + m_j\left(Jgc_pT_j + \frac{V_j^2}{2}\right) = m_e\left(Jgc_pT_e + \frac{V_e^2}{2}\right) \quad (1)$$

Applying the law of conservation of momentum between the inlet and the exit of the mixing tube, friction on the inside surface of the mixing tube being included, and with assumptions 2 and 3, the following equation results:

$$m_aV_a + m_jV_j + p_aA_a + p_jA_j - \Delta p_f\left(A_a + A_j\right) = m_eV_e + p_e\left(A_a + A_j\right) \quad (2)$$

All previous work has been based on the assumption of uniformity of pressure across the mixing-tube inlet $\left(p_a/p_j = 1.0\right)$. The pressure ratio p_a/p_j, however, can have a range of values depending upon the upstream-nozzle design except for the case of both Mach numbers at the inlet being subsonic for which case it is expected that p_a/p_j would be equal to 1.0.

From the law of conservation of mass it is evident that

$$m_a + m_j = m_e \quad (3)$$

Various supplementary equations are needed before the solution of equations (1), (2), and (3) for conditions at the mixing-tube exit can be obtained. From the law for ideal gases the temperature at the mixing-tube eixt can be shown to be

$$T_e = \frac{p_e}{gR} V_e \frac{A_a + A_j}{m_e} \quad (4)$$

Applying the laws of friction in pipes to the mixing tube, the friction pressure drop in equation (2) can be shown to be approximately represented by the following equation:

$$\Delta p_f \cong 2f \frac{l}{d} V_a \frac{m_e}{A_a + A_j} \quad (5)$$

Equation (5) comes from the relation $\frac{\Delta p_f}{q}$ equals $4f \frac{l}{d}$, for which f is empirically determined as $0.049/(Re)^{0.2}$. Equation (5) is an approximation because V_a the velocity of the air at the mixing-tube inlet is used in place of the average velocity of the mixture in the whole tube. The air velocity was used because of lack of knowledge of the mixing process taking place. It was determined that the friction term made little difference in the thrust-augmentation results except for very small tube sizes at extremely high velocities so that such an assumption will lead to little error in most cases.

The Mach numbers at the inlet and exit of the mixing tube can be obtained from the following equations:

$$M_a = V_a\Big/\sqrt{\gamma g R T_a} \tag{6}$$

$$M_j = V_j\Big/\sqrt{\gamma g R T_j} \tag{7}$$

$$M_e = V_e\Big/\sqrt{\gamma g R T_e} \tag{8}$$

The mass rates of flow of air and gas at the mixing-tube inlet can be determined from the equations

$$m_a = \rho_a V_a A_a = \frac{p_a}{gRT_a} V_a A_a \tag{9}$$

$$m_j = \rho_j V_j A_j = \frac{p_j}{gRT_j} V_j A_j \tag{10}$$

The final equations are set up with dimensionless parameters and for convenience the following substitution or change of variable is applied:

$$X = \frac{A_a}{A_j} \tag{11}$$

From equations (1) through (11) and the relation $Jc_p/R = \gamma/(\gamma - 1)$, the static pressure and Mach number at the mixing-tube exit can be determined in terms of conditions at the inlets. The resulting equations are:

$$p_e = \left(p_a\right)\left(\frac{\gamma}{\gamma + 1}\right)\left(\frac{1}{X + 1}\right)\left[A \pm \sqrt{\gamma^2 A^2 - 2(\gamma + 1)B}\right] \qquad (12)$$

$$M_e^2 = \frac{\gamma A \mp \sqrt{\gamma^2 A^2 - 2(\gamma + 1)B}}{\gamma A \pm \gamma\sqrt{\gamma^2 A^2 - 2(\gamma + 1)B}} \qquad (13)$$

where

$$A = XM_a^2 + M_j^2 \frac{p_j}{p_a} + \frac{1}{\gamma}\left(X + \frac{p_j}{p_a}\right) - 2f\,\frac{l}{d}\left(XM_a^2 + M_a M_j \frac{p_j}{p_a}\sqrt{\frac{T_a}{T_j}}\right) \qquad (14)$$

$$B = \left(XM_a + \frac{p_j}{p_a}\sqrt{\frac{T_a}{T_j}}\,M_j\right)\left[XM_a\left(1 + \frac{\gamma - 1}{2} M_a^2\right) + \frac{p_j}{p_a}\sqrt{\frac{T_j}{T_a}}\,M_j\left(1 + \frac{\gamma - 1}{2} M_j^2\right)\right]$$

$$(15)$$

It should be noted that when the plus sign is used in equation (12) the minus sign should be used in the numerator and the plus sign should be used in the denominator in equation (13), and vice versa.

From a knowledge of the static pressure and Mach number at the mixing-tube exit, as determined by equations (12) and (13), respectively, it is possible from the laws of compressible flow to obtain the total pressure at the mixing-tube exit. The relation is

$$\frac{H_e}{p_e} = \left(1 + \frac{\gamma - 1}{2} M_e^2\right)^{\frac{\gamma}{\gamma - 1}} \qquad (16)$$

A curve of Mach number M versus p/H based on this equation is given in figure 3. Also from compressible flow relations

$$\frac{T_{es}}{T_e} = 1 + \frac{\gamma - 1}{2} M_e^2 \qquad (17)$$

On the basis of equations (1), (6), (7), (8), (17), and the relation

$$\frac{Jc_p}{R} = \frac{\gamma}{\gamma - 1} \tag{18}$$

it can be shown that

$$\frac{T_{es}}{T_j} = \frac{1}{\theta + 1}\left[\theta\frac{T_a}{T_j}\left(1 + \frac{\gamma - 1}{2}M_a^2\right) + \left(1 + \frac{\gamma - 1}{2}M_j^2\right)\right] \tag{19}$$

Equations (16), (17), and (19) permit calculation of the required stagnation conditions and temperature at the mixing-tube exit from the pressure p_e and Mach number M_e at the exit and conditions at the mixing-tube inlet. The term θ in equation (19) is the ratio of mass flows of the fluids m_a/m_j and may be simply determined to be

$$\theta = X\frac{M_a}{M_j}\frac{p_a}{p_j}\sqrt{\frac{T_j}{T_a}} \tag{20}$$

In order to show the nature of the solution of equations (12) and (13) consider the trivial case in which (1) the pressure across the mixing-tube inlet is uniform $(p_j = p_a)$, (2) the gas at the jet has undergone neither heat addition nor increase of total pressure, and (3) the flow is frictionless; that is, the same fluid passes through both nozzles and friction is neglected. The net result in the mixing tube is merely the frictionless flow of a fluid in a tube. Equations (12) and (13) then reduce to the common forms given in many references (see reference 6, for example). The solution results in four roots for Mach number at the mixing-tube exit, two of which can be eliminated because they are negative, and two roots of pressure. The two roots of pressure are

$$p_e = p_a \tag{21}$$

and

$$p_e = p_a\left(\frac{2\gamma}{\gamma + 1}M_a^2 - \frac{\gamma - 1}{\gamma + 1}\right) \tag{22}$$

For the condition given by equation (21),

$$M_e = M_a \tag{23}$$

and for that given by equation (22)

$$M_e = \sqrt{\frac{M_a^2(\gamma - 1) + 2}{2\gamma M_a^2 - (\gamma - 1)}} \tag{24}$$

Equations (21) and (23) represent merely the trivial solutions in which the downstream flow is identical with the upstream flow. Equations (22) and (24) will be recognized as the normal shock equations, which obviously also represent a solution because the flow on the two sides of a normal shock fulfill the basic momentum, mass, and energy equations (equations (1), (2), and (3)). In general, solutions of equations (12) and (13) for the ejector nontrivial cases similarly result in four roots for Mach number at the mixing-tube exit, two of which can be eliminated as in the case just noted, the remaining two being related by the normal-shock equations. Further discussion of these two physically significant solutions will be given in a later section of the report.

Conditions at exit of mixing-tube nozzle.- Under conditions to be discussed later, efficiency is improved by use of a subsonic diffuser or a supersonic nozzle (as indicated in figs. 1 and 2) at the exit of the mixing tube. Equations for the conditions at the exit of such a nozzle or diffuser will now be developed.

For purposes of calculation, the nozzle or diffuser at the mixing-tube exit can be assumed to have fixed dimensions for which the static pressure at the nozzle exit p_d will, within limits, be fixed by the upstream conditions, or the pressure p_d can be fixed and the nozzle assumed to be so proportioned that this pressure is obtained for given upstream conditions. The latter method is used in this report. Knowing p_d and if isentropic flow is assumed through the nozzle, the following equations relate conditions at the mixing-tube exit with those at the nozzle exit:

$$M_d^2 = \frac{2}{\gamma - 1}\left[\left(1 + \frac{\gamma - 1}{2} M_e^2\right)\left(\frac{p_e}{p_d}\right)^{\frac{\gamma-1}{\gamma}} - 1\right] \tag{25}$$

and

$$T_d = T_e\left(\frac{p_d}{p_e}\right)^{\frac{\gamma-1}{\gamma}} \tag{26}$$

The determination of the Mach number and temperature at the mixing-tube exit-nozzle discharge thus depends on the temperature, static pressure, and Mach number at the mixing-tube exit and the pressure p_d which is known. ($p_d = p_o$ for highest efficiency.) The equations for M_e and p_e have been presented in the previous section. From the stagnation conditions at the nozzle exit, which are assumed equal to those at the mixing-tube exit, and the Mach number, temperature, and pressure at the nozzle exit, all other conditions at the latter station can be obtained. Equations (16) through (20) can be used to determine the additional conditions at the mixing-tube exit which are required for determining the mixing-tube nozzle-exit conditions.

General Illustrations Showing Effect of Variables on Conditions in Mixing Tube

Application of flight parameters to general equations.- The equations developed in the preceding sections are concerned only with the mixing tube and the mixing-tube nozzle of the ejector, and give the conditions at the mixing tube and mixing-tube nozzle exits in terms of parameters at the mixing-tube inlet. Some of these parameters will now be determined as functions of flight and jet-engine conditions such that the theory will be applicable to flight.

On the basis of assumptions 1 and 4 the following equations can be derived:

$$T_j = T_{us}\left(\frac{p_j}{H_o}\frac{1}{r}\right)^{\frac{\gamma-1}{\gamma}} \tag{27}$$

$$T_a = T_{os}\left(\frac{p_a}{H_o}\right)^{\frac{\gamma-1}{\gamma}} \tag{28}$$

From equations (27) and (28)

$$\frac{T_j}{T_a} = \frac{T_{us}}{T_{os}}\left(\frac{p_j}{p_a}\right)^{\frac{\gamma-1}{\gamma}}\left(\frac{1}{r}\right)^{\frac{\gamma-1}{\gamma}} \tag{29}$$

The stagnation temperature of the free stream T_{os} can be determined from the temperature of the free stream T_o and the Mach number of the free stream M_o by means of an equation similar to equation (17), or

$$T_{os} = T_o\left(1 + \frac{\gamma - 1}{2} M_o^2\right) \tag{30}$$

By means of equations similar to equation (16) applied to the free stream and to the mixing-tube inlet, and again on the basis of assumptions 1 and 4, it can be shown that the Mach numbers of the air and gas at the mixing-tube inlet can be obtained from the equations

$$M_a = \sqrt{\frac{2}{\gamma - 1}\left[\left(\frac{p_o}{p_a}\right)^{\frac{\gamma-1}{\gamma}}\left(1 + \frac{\gamma - 1}{2} M_o^2\right) - 1\right]} \tag{31}$$

$$M_j = \sqrt{\frac{2}{\gamma - 1}\left[\left(\frac{p_o}{p_a}\frac{p_a}{p_j}r\right)^{\frac{\gamma-1}{\gamma}}\left(1 + \frac{\gamma - 1}{2} M_o^2\right) - 1\right]} \tag{32}$$

On the basis of equations (12), (13), (14), (15), (25), (29), (31), and (32) the following equations can be derived:

$$\frac{p_e}{p_o} = \left(\frac{p_a}{p_o}\right)\left(\frac{\gamma}{\gamma + 1}\right)\left(\frac{1}{X + 1}\right)\left[A \pm \sqrt{\gamma^2 A^2 - 2(\gamma + 1)B}\right] \tag{33}$$

$$M_e = \pm\sqrt{\frac{\gamma A \mp \sqrt{\gamma^2 A^2 - 2(\gamma + 1)B}}{\gamma A \pm \gamma\sqrt{\gamma^2 A^2 - 2(\gamma + 1)B}}} \tag{34}$$

$$M_d = \pm\sqrt{\left(\frac{2}{\gamma - 1}\right)\left[\left(1 + \frac{\gamma - 1}{2} M_e^2\right)\left(\frac{p_e}{p_o}\frac{p_o}{p_d}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]} \tag{35}$$

where

$$A = \frac{2}{\gamma - 1}\left\{X(C - 1)\left(1 - 2f\frac{l}{d}\right) + \frac{p_j}{p_a}\left[\left(\frac{p_a}{p_j}r\right)^{\frac{\gamma-1}{\gamma}}(C) - 1\right] + \frac{\gamma - 1}{2\gamma}\left(X + \frac{p_j}{p_a}\right)\right.$$

$$\left. - 2f\frac{l}{d}\frac{p_j}{p_a}\sqrt{\frac{T_{os}}{T_{us}}}\sqrt{(C - 1)\left[\left(\frac{p_a}{p_j}r\right)^{\frac{\gamma-1}{\gamma}}(C) - 1\right]\left(\frac{p_a}{p_j}r\right)^{\frac{\gamma-1}{\gamma}}}\right\} \tag{36}$$

$$B = \left\{X\sqrt{\frac{2}{\gamma - 1}(C - 1)}\right.$$

$$\left. + \frac{p_j}{p_a}\sqrt{\frac{T_{os}}{T_{us}}}\sqrt{\frac{2}{\gamma - 1}\left[\left(\frac{p_a}{p_j}r\right)^{\frac{\gamma-1}{\gamma}}(C) - 1\right]\left(\frac{p_a}{p_j}r\right)^{\frac{\gamma-1}{\gamma}}}\right\}\left\{CX\sqrt{\frac{2}{\gamma - 1}(C - 1)}\right.$$

$$\left. + C\frac{p_j}{p_a}\sqrt{\frac{T_{us}}{T_{os}}}\sqrt{\frac{2}{\gamma - 1}\left[\left(\frac{p_a}{p_j}r\right)^{\frac{\gamma-1}{\gamma}}(C) - 1\right]\left(\frac{p_a}{p_j}r\right)^{\frac{\gamma-1}{\gamma}}}\right\} \tag{37}$$

$$C = \left(\frac{p_o}{p_a}\right)^{\frac{\gamma-1}{\gamma}}\left(1 + \frac{\gamma - 1}{2}M_o^2\right) \tag{38}$$

There are two values of p_e/p_o and two positive values of the mixing-tube exit Mach number M_e which are the only ones used, each of which according to equations (33), (34), (36), (37), and (38) is a function of p_j/p_a, p_a/p_o, T_{us}/T_{os}, r, X, M_o, and fl/d. There are also two positive values of M_d which are the only ones used, each of which depend on the foregoing factors plus p_e/p_o and p_o/p_d.

Thus from flight and jet-engine conditions, the static pressures at the mixing-tube inlet, the area ratio X, the ratio l/d of the mixing tube, and a friction factor, it is possible from the analysis given to determine the static pressure and the Mach number at the mixing-tube exit. In addition to the foregoing parameters, if the static pressure at the mixing-tube and mixing-tube nozzle exits are known, the Mach number M_d at the nozzle exit can be obtained. Graphical illustrations of the effects of the parameters on p_e/p_o and M_e will now be given with discussions of the physically possible solutions. The equations were too complicated to determine analytically the effect of varying one or more parameters, keeping the other parameters constant, on M_e and p_e/p_o. Most of the following study is for frictionless flow because the friction term, as will be illustrated, resulted in a well-defined effect such that results for flow with friction can be approximated closely using frictionless-flow results. The parameter p_j/p_a was arbitrarily set equal to 1.0; that is, the static pressure across the entire inlet of the mixing tube equals p_a for the cases to be discussed. The value of p_a/p_j of 1.0 is used in the following discussion because the effect of variation of p_a/p_j on thrust augmentation, as will be shown later, was not appreciable over the range expected.

<u>Effect of p_a/p_o (frictionless flow; $p_j = p_a$).</u>- The effects of varying p_a/p_o for values of X, r, T_{us}/T_{os}, and M_o of 1.0, 1.0, 1.0, and 1.4, respectively, on the Mach numbers at the inlet and exit of the mixing tube, on the pressure rise through the mixing tube p_e/p_a and on the ratio of static pressure at the exit of the mixing tube to the free-stream static pressure are shown in the curves down the left-hand side of figure 4. In order to vary mixing-tube inlet conditions, it is necessary to change the geometry of the nozzles upstream of the mixing-tube inlet with or without constant mass flow rate of fluid passing through the jet unit when supersonic flow exists at the jet exit. For subsonic flow at the

jet exit, the geometry of the nozzles must change when the mass through the jet unit remains constant in order to vary the ejector inlet conditions. In the present work the mass flow through the jet unit is assumed to be the same and thus p_a/p_o variation of curves like those of figure 4 is accomplished by varying the geometry of the upstream nozzles under the conditions imposed on the curves. The mass rate of flow through the jet unit is assumed the same because the purpose of design is to find the ejector which will give the most thrust increase to a given jet unit. The curves are for frictionless flow fl/d equal to 0 and for the pressure at the jet nozzle exit p_j equal to the pressure at the mixing-tube inlet nozzle exit p_a.

The conditions of flow that will exist in the mixing tube will depend upon the inlet conditions and the conditions downstream of the tube exit. Before determining the flow conditions that will be obtained in flight, a discussion of the flow conditions that will be obtained if the downstream pressure (called the back pressure) can be varied at will is presented. Such a variation of back pressure could be obtained, for instance, by using a tank, in which the pressure was varied, at the ejector discharge in laboratory tests. This discussion will lead to a clearer understanding of the flows obtained in flight. The curves in figure 4 are applicable to such a case. The flight and jet-engine conditions given in figure 4 for this case serve to define the stagnation conditions at the mixing-tube inlet instead of being particular airplane conditions.

For the conditions given for the parameters r, T_{us}/T_{os}, p_a/p_j, and fl/d the curves in the left-hand column of figure 4 revert to the case of frictionless flow of a fluid in a tube. From equations (31) and (32) it is evident that if p_a/p_j and r are equal to 1.0 and M_o a constant value, the Mach numbers at the mixing-tube inlet M_a and M_j are equal and vary only with p_a/p_o variation. This result is shown in figure 4 for the conditions T_{us}/T_{os} of 1.0, one curve representing both the variation of M_a and M_j.

For each value of p_a/p_o and thus Mach number at the mixing-tube inlet, there are two values of Mach number at the mixing-tube exit M_e and two values of the pressure ratio p_e/p_o given in the curves of figure 4 for T_{us}/T_{os} of 1.0. These are the roots previously discussed in the development of the equations. The relations between the inlet and exit pressures and Mach numbers giving rise to these curves are those expressed by equations (21), (22), (23), and (24). For a supersonic flow at the mixing-tube inlet, branches a and b of the curves, both roots are physically significant. One root, branch a, corresponds to the condition of a normal compression shock in the mixing tube with subsonic flow at the exit and will be obtained, except for poorly designed nozzles, when the back pressure equals the mixing-tube exit pressure ($p_b/p_o = p_e/p_o$ where p_b is the back pressure). The other root, branch b, corresponds to the condition of no shock in the tube, supersonic flow being obtained at the mixing-tube exit. From the present theory, for a p_b/p_o equal to p_e/p_o on the branch-a curve at a given p_a/p_o the relation between that back pressure ratio and p_e/p_o on the branch-b curve for the same p_a/p_o value is that for normal compression shock which is a physically possible solution. Thus, for given inlet conditions, two physically possible solutions exist for the same back pressure (that is, the back pressure corresponding to that needed to obtain branch-a result). The same results have been obtained for conditions other than those of the frictionless curve being discussed. This is obviously impossible and experiments show that the back pressure must be decreased slightly to move the shock wave out of a tube until it stands just at the exit. The anomaly in the present theory is caused by the fact that the present theory has been simplified by ignoring the theory of the stability or positioning of the shock wave which involves the boundary layer which would have to be included for completeness. Reference 7 has some work on the stability of shock waves in pipes for those interested. For frictionless flow, no boundary layer exists and, consequently, the shock wave can be at any position in the tube. The stability theory is not applicable and the anomaly of the present theory will always exist for this case. The point is not of great importance because if the physical processes are understood it is not difficult to estimate the actual pressures required to obtain certain flows with probably little error from the present theory. From the foregoing discussion it is evident that the relations between the exit conditions for the two roots for given inlet conditions is that for normal shock. A continued decrease of back pressure below the value for which a normal shock wave stands at the mixing-tube exit will have no further effect on the conditions in the tube but will cause the normal shock wave to change to oblique shock waves with less and less obliquity until a back pressure is reached such that $p_b/p_o = p_e/p_o$ on the branch-b curves. For this condition no shock occurs in or aft of the mixing tube. Further decreases of back pressures below this value result in what are known as rarefaction waves occurring aft of the tube exit.

When p_a/p_o is varied it finally reaches a value where the a and b curves merge into one solution and the Mach number at the mixing-tube exit reaches the value of unity. A decrease of p_b/p_o below the value of p_e/p_o for which the Mach number at the exit is unity will have no effect on the flow in the tube. An increase of p_b/p_o above the foregoing p_e/p_o value for the case of T_{us}/T_{os} of 1.0 in figure 4, will cause the flow to decrease, with no change in nozzle geometry, and subsonic flow to be obtained in the tube. The p_a/p_o value will increase and the d branch of the curves become applicable. For some conditions other than those for frictionless one-fluid flow, a supersonic Mach number at inlet and a Mach number of unity at exit are obtained. If p_b/p_o is increased above p_e/p_o corresponding to this exit Mach number, nozzle geometry remaining the same, a normal shock will occur in the tube, p_a/p_o will decrease, and the a branch of another curve with lower stagnation pressure at the jet unit exit becomes applicable.

When the entrance Mach number is subsonic (branches c and d), only one root of the equation is physically significant, that for subsonic flow at the exit (branch d of the curves, equations (21) and (23)). The other root corresponds to an expansion shock in the tube (equation (24), branch c) with supersonic flow at the exit which is shown to be impossible in reference 8 and other references on the basis of the second law of thermodynamics. The law stipulates that entropy can only increase whereas entropy decreases through an expansion shock. Entropy change thus provided a check as to the physically significant solution of the equations. The branch-c curves have been dashed to denote that the values are only mathematical results. For the branch-d curves, pressure ratio p_b/p_o must equal p_e/p_o to obtain the flow at each p_a/p_o value on the curve except at the point of choking at the mixing-tube exit. If p_b/p_o is less than p_e/p_o for choke at the exit, the conditions in the tube will be the same as when $p_b/p_o = p_e/p_o$ for this condition. For complete subsonic flow in the mixing tube the flow depends directly upon the back pressure. If the back pressure changes from a value giving a flow at a certain p_a/p_o value and the nozzle geometry is not changed, the mass of fluid flowing will change until a new set of inlet conditions are obtained such that the p_e/p_o ratio for these conditions will equal the new p_b/p_o ratio.

The foregoing discussion has dealt with the case of a varying back pressure with the ejector discharging into a hypothetical tank. In flight, a field of fluid with a Mach number M_o and a pressure p_o, the atmospheric pressure, surrounds the jet discharging from the ejector. As this was the problem of most interest, the inlet and exit pressures in figure 4 have been expressed as ratios of p_o. The flow conditions in the mixing tube that will exist in flight depend upon the mixing-tube inlet conditions and the back pressure p_o if the flight speed is subsonic $(M_o < 1.0)$ but will only depend upon the mixing-tube inlet conditions if the flight speed is supersonic.

For subsonic flight speeds, the discussion for effect of back pressure in a tank on the flow conditions that will be obtained is directly applicable for determining the flow with a pressure p_o aft of the exit. With subsonic flow at exit, the only physically obtainable solutions on the a and d branches of the curves are those for which $p_e/p_o = 1.0$. If p_a/p_o is any value other than those for which $p_e/p_o = 1.0$ on the a or d branches, one of two conditions will result:

1. For p_e/p_o not necessarily equal to 1.0 and supersonic flow at the inlet, the resulting flow at the exit will be supersonic and branch b of the curves applicable. The shock conditions existing aft of the exit will depend upon the p_e/p_o obtained for the existing inlet conditions as previously discussed. The b branch of the curves is applicable for a variation of p_e/p_o from values greater than 1.0 down to the value where a normal shock stands just outside the exit. On the basis of previous analysis this latter value would be at the same ratio of pressure at inlet to free-stream pressure p_a/p_o for which $p_e/p_o = 1.0$ on the a branch of the curves.

2. If the jet nozzles are designed with the expectation of a certain subsonic flow at the inlet (d branch of the curves) and, consequently, a certain p_a/p_o value and flow rate which would

result in a pressure ratio p_e/p_o not equal to 1.0, the flow rate that will actually be obtained with these nozzlos will be such that p_e/p_o will equal 1.0 with subsonic or sonic flow at the exit or p_e/p_o will bo greater than 1.0 with sonic flow at the exit.

For supersonic flight speeds, the pressure at the exit of the mixing tube can be greater or less than p_o for both subsonic and supersonic flow at the mixing-tube exit. The reason p_e/p_o does not have to be equal to 1.0 is the same as that for which p_a/p_j doos not have to be equal to 1.0 if one or both of the mixing-tube inlet Mach numbers is supersonic. The prossure p_e adjusts aft of the exit to the pressure p_o by different types of shock waves, those for supersonic flow at the exit having been described. Consequently, all solutions on the a, b, and d branches of the curves can be obtained regardless of the ratio of the exit pressure to the atmospheric pressure. The anomaly obtained from the theory that two flow conditions are possible, one on the a branch and one on the b branch, for one back pressure in a tank also exists for the flight case. It exists over the whole range of the a and b branches when the flight speed is supersonic but cnly at one point on these branches when the flight speed is subsonic. This point is at the p_a/p_o for which $p_e/p_o = 1.0$ on the a branch. As explained previously, this is of no practical consequence, the flow conditions that will be obtained being able to be determined.

Effect of T_{us}/T_{os}.- The effect of heating of the gas on the exit conditions is obtainod from figure 4 by comparing the curves for the three T_{us}/T_{os} values of 1.0, 1.55, and 3.16. The inlet Mach number curve obtained for T_{us}/T_{os} of 1.0 is applicable to the other temperature ratios because, as shown by equations (31) and (32), the inlet Mach numbers are not affected by the parameter T_{us}/T_{os}. For a T_{us}/T_{os} of 1.0, all branches of the exit Mach number curves reach unity at a common value of inlet pressure to free-stream pressure ratio. This point of joining of the curves becomes two points when the temperature ratio exceeds 1.0, the range of p_a/p_o between the two points increasing as T_{us}/T_{os} increases as indicated in figure 4. Thus, as p_a/p_o is increased proceeding along the a and b branches, a value of p_a/p_o is reachod for which the exit Mach number for both branches is equal to one and the branches join. A gap then exists for which no solutions were obtained. Then a p_a/p_o value is reached where the branches c and d meet and a Mach number of unity at exit is again obtained. Further increase in the p_a/p_o values gives solutions to the equations. For supersonic flow at the inlet, the greater the amount of heating the lower the value of p_a/p_o and the higher the value of the Mach number at the inlet will be when choking occurs at the exit. For subsonic flow at the inlet, the higher the value of p_a/p_o and the lower the value of the inlet Mach number will be when choking occurs at the exit for greater heating. The relations between the Mach numbers M_e and the pressure ratio p_e/p_o on the a and b branches of the curves at a given p_a/p_o value are the normal-shock relations as in the case of the T_{us}/T_{os} of 1.0 curves.

The reason for the gap in the p_a/p_o values for which no solutions are obtained is that attempts to obtain a p_a/p_o value infinitesimally higher than that at the lower limit, for which M_e is 1.0 by a suitably designed nozzle, causes a normal shock at the exits of the inlet nozzles with the Mach number upstream of the shock equal to M_a at the lower limit p_a/p_o value and the Mach number downstream of the shock being equal to M_a at the upper limit p_a/p_o value, the relation between the lower and upper limit inlet Mach numbers being that for normal shock as determined for both T_{us}/T_{os} of 1.55 and 3.16 in figure 4. But this latter Mach number results in the choking exit Mach number also, which is the result required from the physical standpoint. Upstream nozzles, designed for expected p_a/p_o values between the lower limit value and that for M_a of 1.0, will have shocks moving progressively upstream from the nozzle exits as expected p_a/p_o increases, the positioning of the shocks always being such as to result in the subsonic Mach numbers at the nozzle exits, or mixing-tube inlet, equal to those at the upper p_a/p_o limits of figure 4 curves.

Efforts to change p_a/p_o to values lower than that for which the Mach number equals 1.0 at the mixing-tube exit, with subsonic flow at the inlet, and keep the mass flow rate through the jet unit unchanged by changing the geometries of the upstream nozzles (that is, their exit areas) result in a decrease in the mass fluid flowing, as compared to that flowing at the upper limit, the pressure ratio p_a/p_o being equal to that at the upper limit line and the inlet Mach number remaining the same as its value at the upper limit line.

The c branches of the curves in figure 4 for temperature ratios greater than 1.0 have been dotted to denote conditions which are thought to be impossible to obtain by reasoning based on the results for a temperature ratio of 1.0. No proof that the conditions are not obtainable could be derived from a calculation of the entropy change between the inlet and exit of the mixing tube. All calculations for these results and all subsequent results but one gave positive entropy changes. It is known that when two gases mix the entropy increases. This action is occurring in the mixing tube but because of lack of knowledge of the mixing process a step by step calculation of entropy change could not be made. It is probable that the over-all positive entropy change from inlet to exit is composed of entropy change due to mixing which is positive and an entropy change from expansion shock which would be negative.

Effect of pressure-rise ratio r of actuating jet.- Sets of curves similar to those in figure 4 are given in figure 5 to illustrate the effect of pressure-rise ratio of the actuating jet on mixing-tube conditions for the case of no heat $(T_{us}/T_{os} = 1.0)$ and no friction. The results are shown for three ratios, r, of 1.0, 6.0, and 12.0. The conditions of figure 5 are such that the case of r equal to 1.0 duplicates the case of one-fluid flow in a tube given in figure 4, the basic case for comparison with other cases, and the curves are identical to those in the latter figure.

For pressure-rise ratios greater than 1.0, the Mach numbers M_a and M_j at the mixing-tube inlet are not equal. The Mach number M_a at ratios of 6 and 12 in figure 5 is equal to that at a ratio of 1.0, for a given p_a/p_o value, but the Mach number at the jet-nozzle exit M_j increases above the value at a pressure-rise ratio of 1.0 as the ratio increases. This is to be expected on the basis of equations (32) and (33). The general trend of conditions at the mixing-tube exit as the pressure-rise ratio increases is away from the choking conditions. For the conditions of figure 5, as r increased, continuous solutions over the range of p_a/p_o were obtained, the branches a and d joining to form a continuous curve and the branches c and d doing the same. No choking at exit was obtained at r of 6.0 and 12.0. The effect of pressure-rise ratio on the conditions at the exit of the mixing tube is thus opposite to the effect of the temperature ratio T_{us}/T_{os}.

Combined effect of pressure-rise ratio r and heating of actuating jet.- Figure 6 shows curves similar to those of figure 5 for pressure-rise ratios r of the jet engine of 1.0 and 6.0 but with heat addition in the jet engine equivalent to the highest value used in figure 4 (T_{us}/T_{os} of 3.16). The curves are for frictionless flow and for an area ratio X at the mixing-tube inlet of 3.0 rather than 1.0 as in figures 4 and 5. The purpose of the calculations of figure 6 was to determine whether the effect of r on the mixing-tube exit conditions would overcome the counteracting effect of heat to such an extent as to eliminate the choking action of heat obtained in figure 4. The value of X of 3.0 was used because it was considered a reasonable value for a turbojet installation. For the pressure-rise ratio of 1.0 in figure 6, the curves show the same type of solution obtained with heat in figure 4, limiting values of inlet Mach number being obtained for which the mixing-tube chokes at the exit. For a range of p_a/p_o values no solution was obtained. Raising the pressure-rise ratio to 6.0, however, eliminated the conditions of shock or falling off of flow in the upstream nozzles which result from choking conditions as exist at r equal to 1.0, continuous solutions and no choking being obtained over the whole p_a/p_o range.

The range of the c branch of the curves for the case where the entrance Mach numbers M_a and M_j are not equal and where the curves are continuous is impossible to obtain mathematically from the present theory. The method of calculating the entropy change, which for the c branch of the curve is negative and for the d branch is positive in the case of one-fluid flow, cannot be used because positive values are obtained for most cases as explained. The point of intersection of the c and b branches of the curves for cases as shown in figure 6 for r equal to 6.0 has been taken as the lowest point of the M_e versus p_a/p_o curve. This point was considered logical as both the c and d branches are tending to reach the point where the exit Mach number is unity as shown in figure 5 for a pressure-rise ratio of 1.0. The range of p_a/p_o over which the d branch extends is the same as that for the c branch.

Effect of area ratio X at mixing-tube inlet.- The effect of X, the ratio of the area at the mixing-tube inlet through which the secondary air passes to the area of the jet nozzle exit, on the conditions at the mixing-tube exit has the same trend as the effect of heat; that is, increase of X causes the Mach number at the exit to approach the value of unity for given inlet conditions.

This trend is shown in the curves of figure 7. The conditions of the curves of figure 7 are for a medium pressure-rise ratio, high heat, and frictionless flow. The conditions in the tube are shown for area ratios X of 1.0, 3.0, and 6.0. The curves for X equal to 3.0 are identical to those on figure 6 for r of 6.0. Figure 7 shows that even with an X of 6.0, the effects of X and high heat tending to cause choking at the exit were not great enough to overcome the effect of r the tendency of which is to cause the Mach number at exit to recede from the value of one as r increases. With an X of 6.0, continuous solutions were obtained and a Mach number of one at the mixing-tube exit was not reached although it was approached very closely.

Effect of free-stream Mach number M_o.- Curves of the type given can be easily obtained for any free-stream Mach number, area ratio X, temperature ratio T_{us}/T_{os}, and pressure-rise ratio r (the ratio $p_j/p_a = 1.0$) when curves are available for one free-stream Mach number and for values of the other parameters equal to those for which curves are required. From equations (12), (13), (14), (15), (29), (31), and (32) it can be deduced that for the same value of M_a, M_j, M_e, or p_e/p_a on the given and required curves the ratio of the abscissas of the curves will be given by the expression:

$$\frac{\left(\frac{p_a}{p_o}\right)_{required}}{\left(\frac{p_a}{p_o}\right)_{given}} = \left[\frac{1 + \frac{\gamma - 1}{2}\left(M_o\right)^2_{\ required}}{1 + \frac{\gamma - 1}{2}\left(M_o\right)^2_{\ given}}\right]^{\frac{\gamma}{\gamma-1}}$$

The required p_e/p_o versus p_a/p_o curves can be obtained from the required p_e/p_a versus p_a/p_o curves obtained as described by multiplying the values of p_e/p_a by the corresponding values of p_a/p_o and plotting the resulting products against the corresponding p_a/p_o values.

Effect of friction.- The curves of figures 4, 5, 6, and 7 were for frictionless flow. The effect of friction is illustrated in figure 8 for one set of values of the parameters: $X = 1.5$, $M_o = 0.8$, $T_{us}/T_{os} = 2.66$, and $r = 1.5$. From the curves of figure 8 it is evident that friction causes the exit Mach numbers to approach the value of one in a manner similar to the action of heat and increase of area ratio X. The range of p_a/p_o for which no solution was obtained in figure 8 increased appreciably for the case of friction as compared with the case of no friction. At the p_a/p_o values, however, for which solutions are obtained, the values of the mixing-tube exit conditions without friction change very little for the subsonic branches of the curves (a and d) when friction was considered. The values for the supersonic branch (branch b) showed more change.

Résumé

The work that has been presented represents a general treatment of compressible flow in an ejector with a derivation of basic relations for conditions in the mixing tube and a discussion with illustrations and explanations of the effects of various parameters on the conditions. The basic relations generally involved the Mach numbers of the flow because such treatment simplified the theory appreciably. From such a general treatment, it is possible to apply the work to specific problems involving the ejector. The foregoing sections complete the major object of the present report.

As an example of application of the general treatment, the next section of the report takes up the problem of the theoretical thrust augmentation obtained using ejectors on jet-propelled airplanes and missiles over wide ranges of free-stream Mach numbers. This was a secondary object of the report as mentioned in the Introduction. The direct application of the equations developed in the general treatment will be readily perceived.

APPLICATION OF GENERAL TREATMENT TO THRUST AUGMENTATION

The thrust of any device is equal to the rate of change of momentum of the fluid involved between stations of equal pressure upstream and downstream. Momentum, of course, is equal to mass of fluid involved multiplied by velocity of the fluid. In the present problem, the case of thrust augmentation with the pressure at the mixing-tube exit equal to the free-stream pressure $(p_e/p_o$ equal to 1.0) is the only one considered to any extent for ejectors with no mixing-tube exit nozzle. For the case of ejectors with mixing-tube exit nozzles with discharge from the nozzle to the atmosphere, the only thrust-augmentation results presented are those for the pressure at the nozzle exit equal to free-stream pressure (p_d/p_o equal to 1.0). The following thrust equations are derived on the basis that all discharge pressures are equal to the free-stream pressure. The thrust effects of small amounts of fuel are neglected.

Thrust Augmentation Equations

Thrust without ejector.- The thrust for an airplane without an ejector traveling with a velocity V_o, with a velocity V_{jF} at the jet-nozzle exit, and with the discharge pressure equal to free-stream pressure is given by the equation

$$\text{Thrust} = m_{jF}\left(V_{jF} - V_o\right) \tag{39}$$

The subscript jF denotes free jet.

Thrust with ejector.- The thrust of an airplane with an ejector with no mixing-tube exit nozzle and with discharge from the mixing-tube exit at velocity V_e and pressure p_o is given by the equation

$$\text{Thrust} = \left(m_a + m_j\right)\left(V_e - V_o\right) \tag{40}$$

Equation (40) can be used for the case of an ejector with a mixing-tube exit nozzle from which the fluid discharges at velocity V_d and pressure p_o by replacing V_e in the equation with V_d.

Thrust ratio.- The ratio of the thrust with the ejector to the thrust of the same jet unit with the same operating conditions without the ejector ϕ can be obtained from equations (39) and (40). For the purpose of this report the ratio m_j/m_{jF} was assumed equal to unity. This seemed the most practical case to calculate. The attainment of this condition usually requires that the size and possibly the shape of the actuating jet nozzle be changed when the ejector is added. With this assumption and with a constant heat addition with and without the ejector, the value of ϕ is not only a thrust ratio but is also an efficiency ratio. A 20-percent increase in thrust, for instance, would also signify a 20-percent increase in over-all efficiency. If ϕ is greater than 1.0, thrust augmentation is obtained. If ϕ is less than 1.0 and positive, thrust with the ejector is less than the thrust without the ejector. If ϕ is less than zero (negative), the addition of the ejector creates drag instead of thrust.

From equations (39) and (40), some equations of the general theory, and the foregoing discussion it can be easily shown that the thrust ratio using only a straight mixing tube can be obtained from the following equation in which the parameters are dimensionless:

$$\phi = (\theta + 1)\frac{M_e\sqrt{\frac{T_e}{T_o}} - M_o}{M_{JF}\sqrt{\frac{T_{JF}}{T_o}} - M_o} \tag{41}$$

From equations (26) and (29), the term θ in equation (41) becomes equal to

$$\theta = X\frac{M_a}{M_j}\left[\sqrt{\frac{T_{us}}{T_{os}}\left(\frac{1}{r}\right)^{\frac{\gamma-1}{\gamma}}}\right]\left(\frac{p_a}{p_j}\right)^{\frac{\gamma+1}{2\gamma}} \tag{42}$$

The term p_a/p_j drops out of equation (42) if uniformity of pressure at the mixing-tube inlet is assumed. In like manner from preceding relations (equation (23)), equations similar to equation (23) for the air and gas at the mixing-tube inlet, and equations (25) and (30), it can be shown that

$$\frac{T_e}{T_o} = \frac{1 + \frac{\gamma - 1}{2}M_o^2}{1 + \frac{\gamma - 1}{2}M_e^2}\left(\frac{\theta}{\theta + 1} + \frac{T_{us}}{T_{os}}\frac{1}{\theta + 1}\right) \tag{43}$$

For the case of the free jet, equation (32) is applicable for determining M_{JF}. For the case of p_{JF} equal to p_o, which is the condition for which equation (41) is applicable, equation (32) becomes:

$$M_{JF} = \sqrt{\frac{2}{\gamma - 1}\left[(r)^{\frac{\gamma-1}{\gamma}}\left(1 + \frac{\gamma - 1}{2} M_o^2\right) - 1\right]} \tag{44}$$

On the basis of assumptions 1 and 5, and the relations between stagnation and static pressures and stagnation and fluid temperatures at a section, the temperature ratio T_{JF}/T_o in equation (41) reduces to

$$\frac{T_{JF}}{T_o} = \frac{1}{(r)^{\frac{\gamma-1}{\gamma}}} \frac{T_{us}}{T_{os}} \tag{45}$$

The thrust ratio with a nozzle at the mixing-tube exit is determined by an equation similar to equation (41) with M_e and T_e/T_o replaced with the terms M_d and T_d/T_o, respectively. The Mach number at the mixing-tube nozzle exit M_d is determined by means of equation (35) using the positive root. The temperature ratio T_d/T_o can be obtained from the following equation on the basis of assumption 1

$$\frac{T_d}{T_o} = \frac{T_e}{T_o}\left(\frac{p_d}{p_e}\right)^{\frac{\gamma-1}{\gamma}} \tag{46}$$

In order to determine the thrust augmentation of an ejector for the particular cases of $p_e/p_o = 1.0$ when no nozzle is used or $p_d/p_o = 1.0$ when a nozzle is used, the following parameters must be known:

(a). Flight Mach number, M_o

(b) Jet engine pressure-rise ratio, r

(c) Stagnation temperature ratio, T_{us}/T_{os}

(d) Area ratio at mixing-tube inlet, X

(e) Length-diameter ratio of mixing tube, l/d

(f) Friction factor in mixing tube, f

(g) Pressure ratio at mixing-tube inlet, p_a/p_j

(h) Pressure ratio of air at mixing-tube inlet, p_a/p_o (value for the condition p_e/p_o of 1.0 for the case of no nozzle or p_d/p_o of 1.0 for the case with a nozzle, must be known)

It should be noted that the omission of the friction term involving the parameter f from the mixing-tube calculations, which generally affected the absolute values of thrust augmentation very little, might lead to distortion of the trends of the different parameter effects especially with small mixing tubes in which the velocities are extremely high.

The following section gives the detail method of solving the equations given heretofore to determine the thrust augmentation for given values of the parameters just enumerated.

Method of Solving Equations

Ejector with mixing tube discharging to atmosphere.- Values of M_o, r, T_{us}/T_o (if T_{us}/T_o and M_o are known, T_{us}/T_{os} is easily determined), X, l/d, f, and p_a/p_j are first assumed. Reference to the preceding list of parameters shows that the remaining parameter required before thrust augmentation can be calculated is p_a/p_o. In this problem the values of p_a/p_o for which $p_e/p_o = 1.0$ were the only ones used in the thrust determinations. From a study of the general illustrations of figures 4 through 8 it can be determined that one, two, or three values of p_a/p_o will result in the condition p_e/p_o of 1.0 depending on the values of the fundamental parameters. When three roots are obtained, the highest value falls on the branch c of the curves that is considered not significant physically. This point is illustrated in figures 4 through 7 at p_e/p_o of 1.0. Determination of the values of p_a/p_o for which $p_e/p_o = 1.0$ for each assumed set of the remaining seven parameters then constitutes the second step of the solution. This is accomplished graphically by drawing curves similar to those of figure 6. The branch of the curve on which each p_a/p_o value falls is noted. These parameters are then used in further calculations to determine M_e using equation (34) (positive root), θ using equation (42), T_e/T_o using equation (43), M_{JF} using equation (44), and T_{JF}/T_o using equation (45). All terms for obtaining the thrust ratio ϕ by means of equation (41) are then available. In the curves of ϕ to be presented, notations on the branches (a, b, c, or d) correspond to the notations on the branches from which the p_a/p_o values were obtained.

<u>Ejector with discharge to atmosphere from nozzle at mixing-tube exit.</u>- When an ejector with a nozzle at the exit of the mixing tube is used, only the case of the nozzle-exit pressure equal to the atmospheric pressure $(p_d/p_o = 1.0)$ is considered as previously explained. For this case it would be supposed that any value of p_a/p_o could be assumed and the resulting value of the pressure ratio p_e/p_o could be converted to a pressure ratio p_d/p_o of 1.0 at the nozzle exit by proper proportioning of the nozzle. In general, such conversion is always theoretically possible provided only that H_e exceeds p_o. Low values of p_a/p_o sometimes result in this condition. Certain other values will not give solutions because they are in a range between two values of p_a/p_o that result in choking Mach numbers at the mixing-tube nozzle exit, the phenomena occurring which prevents such p_a/p_o values being possible having been explained in the general treatment for the case of choking occurring at the mixing-tube exit. All other values of p_a/p_o will give solutions and for each value the same calculations are made as in the case of no nozzle with the exception of the calculation of ϕ by means of equation (41). Additional calculations of M_d (equation (35), positive root), and T_d/T_o (equation (46)) are

required before the thrust ratio can be calculated. The term p_d/p_e in equations (35) and (44) is replaced with (p_d/p_o) (p_o/p_e). The value of p_d/p_o is then equal to 1.0 and p_e/p_o is determined during the course of the calculations as explained.

The curves of thrust ratio ϕ versus p_a/p_o for the case of a nozzle at the mixing-tube exit will have branches corresponding to solutions as shown in the general treatment illustrations. Each p_a/p_o value requires particular nozzles. No attempt has been made to determine nozzle proportions. For those interested in this phase of the problem, formulas are available in many sources, among them reference 9.

Conditions Used in Calculations

The conditions that had to be selected and for which the calculations of thrust ratio were made were, as noted previously: the Mach number of flight M_o, pressure-rise ratio of the jet unit r, the upstream stagnation temperature ratio T_{us}/T_{os}, the area ratio X, the ratio of mixing-tube inlet static pressures p_a/p_j, ejector geometry, and the friction factor. The ranges of conditions used for the calculations for turbojet and ram-jet units and the reasons for the choices are given in the following discussion.

Flight Mach number M_o.- As most investigators had obtained good thrust at the static condition, but the work of Howell and of Slatter and Bailey in England had shown a rapid decrease of thrust augmentation as the airplane velocity increased, most calculations of thrust ratio for turbojets with an ejector without a nozzle at the mixing-tube exit were made over a range of flight Mach number M_o from 0 to 1.4. Calculations for ejectors with mixing-tube exit nozzles were made for a typical airplane flight Mach number of 0.8 and for a typical missile flight Mach number of 2.0. The calculation for ram-jet units with ejectors both with and without mixing-tube exit nozzles were made for flight Mach numbers of 1.4, 2.0, and 3.0.

Pressure rise and upstream stagnation temperature ratios (r and T_{us}/T_{os}).- The effect of altitude, airplane velocity, and jet-unit downstream pressures and temperatures are combined in the parameters r, T_{us}/T_o, and M_o. The temperature ratio T_{us}/T_{os}

depends upon T_{us}/T_o and M_o. The pressure-rise ratio r of the jet unit and the temperature ratio T_{us}/T_o were varied over what are considered practical values of turbojets at present to values now being considered (r from 1.5 to 12.0; T_{us}/T_o from 3 to 9) for the calculations for ejectors with no mixing-tube exit nozzle. For the calculations for ejectors with the nozzle, the extreme values of r and T_{us}/T_o given in the preceding statement were used.

For the calculations using ram-jet units, r was assumed equal to 1.0; that is, no pressure drop across the unit would be obtained. The T_{us}/T_o ratio for these units was obtained by assuming the air-fuel ratio. An air-fuel ratio of 15 was chosen for the ram-jet calculations with and without the ejector with a flow rate through the jet unit equal to m_j. This value, which is near the stoichiometric mixture, was chosen because there is little doubt that the maximum thrust of ram jets will be obtained in the stoichiometric mixture range. The practical side of the possible use of the high temperatures that will be obtained at low air-fuel ratios is not considered pertinent to a theoretical report. The term T_{us}/T_o for the ram-jet calculations equals $(T_{os}/T_o) + (\Delta T/T_o)$. The first ratio was determined from the flight Mach number using equation (30). The temperature T_o was obtained from standard altitude tables (reference 10) for an altitude of 30,000 feet. The temperature rise ΔT was derived from chemical combustion analysis knowing the air-fuel ratio.

Area ratio X.- A ratio of area through which the secondary air flows at the mixing-tube inlet to the jet-nozzle exit area X of 1.5 was used for most of the calculations for ejectors with no mixing-tube exit nozzle in combination with turbojet units but a range of X values was used with the extreme values of r and T_{us}/T_o. A ratio of areas of 10, which is considered impractical from a construction standpoint, was also used for the static condition only to determine the trend of thrust ratio with area ratio past the practical range. Values of X of 1.5 and 6.0 were used in the calculations for ejectors with mixing-tube exit nozzles in combination with turbojet units. The latter two values were also used in the calculations for ejectors both with and without mixing-tube exit nozzles in combination with ram-jet units.

Mixing-tube inlet static pressure ratio p_a/p_j.- The pressure ratio p_a/p_j as brought out in the theory, can have a range of values depending upon the upstream-nozzle design except for the case of both Mach numbers at the inlet being subsonic for which case it is expected that p_a/p_j would be equal to 1.0. For a given set of parameters other than p_a/p_o and p_a/p_j, there are numerous combinations of p_a/p_o and p_a/p_j for each branch of a set of curves like figure 6 that will result in a p_e/p_o of 1.0. For the purpose of the present problem, all the combinations of p_a/p_j and p_a/p_o that result in a p_a/p_o of 1.0 were not used to obtain the thrust ratios. A pressure ratio p_a/p_j equal to 1.0 was used for almost all calculations, which simplified the problem appreciably. As noted, this is the only ratio that will probably exist when M_a and M_j are both less than 1.0. One set of calculations were made, however, varying p_a/p_j from 0.2 to 1.4 at a low and high free-stream Mach number to determine the effect of this parameter on the thrust ratio.

Ejector geometry.- The mixing tube in all cases was assumed to be a cylinder. A length to diameter ratio l/d of 10 was used in all calculations because experiments had indicated that ratios of this order of magnitude or greater were needed for complete mixing, on which basis the theory was set up. By using a mixing tube with no nozzle at its exit, it was recognized that inferior results would be obtained in some cases due to shocks developing in the tube. It was considered a logical first step, however, to explore the possibilities of such a simple device as regards thrust augmentation. The logical second step, which was made, was to determine the possibilities of ejectors with nozzles at the mixing-tube exit. The nozzle geometries for the required conditions were not determined, as explained previously, because they were not pertinent to the purpose of the problem.

Friction factor.- A constant value of friction factor f of 0.0035 was used throughout the calculations. This value represented an average of the values for the flow conditions and changing it changed the end results very little.

The majority of the results worked up were for the case of ejector-turbojet combinations. A few conditions were chosen, however, so as to obtain a brief study of the effect of the addition of an ejector, with and without a nozzle at the mixing-tube exit to an airplane or missile with a simple ram-jet unit from a thrust standpoint. A secondary purpose of such an installation is the efficacy of the secondary air for cooling purposes when such cooling is required for structural reasons for supersonic airplanes and missiles. It is possible that a simple ram jet occupying the same space and using air-flow quantities equal to the sum of the actuating air and the secondary air and fuel quantities equal to those of the ejector-equipped ram jet would give more thrust and have a better efficiency than the latter unit, although the former unit would not perform the cooling function mentioned. A few simple comparisons were made to determine this effect. The ratio of thrust of the large ram jet with an air-flow rate equal to $(m_a + m_j)$, and the small ram jet with an air-flow rate equal to m_j, both without an ejector and with the same free-stream velocity, was determined from the formula:

$$\phi = \frac{(\text{Over-all efficiency} \times \text{Heat input})_{\text{Large ram jet}}}{(\text{Over-all efficiency} \times \text{Heat input})_{\text{Small ram jet}}}$$

The ideal over-all efficiencies were obtained using a formula given in reference 11 which involved the combustion temperature rise ΔT and some of the conditions given in the table for the ram-jet cases. The combustion temperature rise was determined from the fuel-air ratio by the method given previously. For the large ram jet, the air-fuel ratio was determined using the same fuel quantity as the small ram jet and the large air quantity. The mass of air flowing through the ram jet times the product of c_p and ΔT gave the heat input. The ideal case of no loss across the burner was used for all ram-jet calculations.

Laboratory tests under static conditions indicated good thrust augmentation with an ejector (reference 5). Most of these tests were made with compressed air and with very great ranges of mixing-tube inlet-area ratios. Consequently, conditions were chosen to simulate some of those in reference 5 as closely as could be ascertained from this reference in order to determine whether the large thrust gains obtained experimentally were indicated by the theory and if the use of compressed air instead of hot air as the actuating gas had a beneficial effect on the thrust.

The sets of conditions for which the calculations of thrust ratio were made and which were discussed in the foregoing sections are listed in table I at the end of the report.

Thrust Augmentation of Turbojet Units

<u>Mixing tube discharging to atmosphere ($p_e/p_o = 1.0$)</u>.- The thrust augmentation for a range of flight Mach numbers from 0 to 1.4 with a low ratio of secondary-air area to area of jet-nozzle exit at the mixing-tube inlet ($X = 1.5$) for several pressure-rise ratios and heat inputs represented by T_{us}/T_o of the actuating jet is given in figure 9. All calculations for this figure are for the conditions p_e/p_o and p_a/p_j equal to 1.0. Figure 9(a) gives results at four pressure-rise ratios with a low heat input (T_{us}/T_o, 3.0). Figure 9(b) and 9(c) give similar results for T_{us}/T_o values of 6.0 and 9.0, respectively. Sections of the curves are labelled a, b, c, and d in order to show which branch of curves similar to those of figures 4 to 8 each point of the curves of figure 9 will fall on for the condition $p_e/p_o = 1.0$.

From the curves of figure 9(a) it is evident that for low flight Mach numbers and pressure only one point in a set of curves such as those of figure 4 would have a value of p_e/p_o of 1.0 and it would be on the d branch. As the flight Mach number increases, the velocity through the ejector increases until the condition p_e/p_o of 1.0 is obtained for three values of p_a/p_o (on a, b, and c branches of curves). This result gives three values of thrust ratio as shown in figure 9(a) for r of 1.5 at high flight Mach numbers. The reasons for the curves at other conditions in figure 9 having the shape given can be derived from the general illustrations in like manner. The points of intersection of branches c and d of some curves of figure 9 and subsequent figures is the point of choking at the mixing-tube exit.

A study of figure 9 shows that generally thrust augmentation is obtained only at very low flight Mach numbers, although for one set of conditions ($r = 6.0$, $T_{us}/T_o = 9$; figure 9(c)) this does not even occur. The d-c continuous curves have the downward then upward trend obtained by others as flight Mach number increases, but for the most part at high Mach numbers, thrust augmentation is shown only by the c branch which has been considered physically not possible. Turbojet engines with low heat input appear to benefit the most by using the ejector. At low flight Mach numbers the low pressure-rise ratio engine is benefitted more by using the ejector than the high-ratio engine. At high Mach numbers the thrust of the engine without the ejector was decreased less when the ejector was used at high pressure-rise ratios than at low ratios. The greatest thrust augmentation obtained for all conditions shown in figure 9 was about 7.5 percent for the static conditions with an r of 1.5 and a T_{us}/T_o of 3.0. This thrust increase decreased rapidly as the flight Mach number increased until at M_o of about 0.17 the thrust with the ejector was equal to the thrust without the ejector.

Over small ranges of M_o in figure 9 for some conditions only the thrust ratio for the c branch of the curve is obtained which is considered not physically significant. The conditions that are expected to be obtained in the mixing tube for such flight and actuating-jet conditions are those on the d branch with p_e/p_o being other than 1.0 and depending on the flight Mach number. For this result, a thrust-ratio formula other than that given would be required to calculate the thrust augmentation.

Curves similar to those of figure 9 are shown in figure 10 for the extreme values of pressure-rise ratio and temperature ratio used in figure 9 and for area ratios X of 3.0 and 6.0. The same trends with variation of M_o, r, and T_{us}/T_o noted in figure 9 for X of 1.5 are apparent in figure 10 for the higher area ratios. In general, thrust augmentation is obtained only at very low flight Mach numbers but the values are a little higher than those of figure 9. Thus at X of 6, r of 1.5, and T_{us}/T_o of 3.0 the thrust ratio for the static condition is a little greater than 1.2. (See fig. 10(a).) At the low pressure-rise ratio and high temperature ratio (fig. 10(a)) some thrust augmentation is obtained at values of M_o from about 0.7 to 1.0; at the latter Mach number about 10 percent increase in thrust being obtained for area ratios of 3.0 and 6.0. The latter condition is an isolated instance, no thrust increase being obtained for a greater part of the range of M_o in figure 10.

The results of figures 9 and 10 have been cross plotted in figure 11 to determine the effect of area ratio X on thrust ratio for four constant flight Mach numbers (M_o of 0, 0.6, 1.0, and 1.4). The thrust-ratio scales are very small but the general picture of effect of X for extreme values of pressure-rise ratio and temperature ratio was desired in one figure for easy comparison of trends rather than exact values of thrust ratio. It is very evident from figure 11 that for the flight Mach numbers chosen, thrust augmentation is obtained only at the static condition, the greatest amount being obtained at r of 1.5 and T_{us}/T_o of 3.0.

The curve for the static condition for r of 1.5 and T_{us}/T_o of 3.0 in figure 11 has been replotted to a larger scale in figure 12, together with the thrust ratio for an area ratio of 10 and the same conditions of r and T_{us}/T_o. This curve was drawn to obtain a better idea of the thrust increases and to ascertain whether the increasing trend of thrust ratio with increase of X continued past X of 6.0. Figure 12 shows that the thrust ratio continues to increase over the range of X shown being 1.075 at the lowest point and 1.28 at the highest point. The appreciable increases noted do not persist, as illustrated previously, at high flight speeds. For low pressure-rise ratio and high heat input it can be deduced from figures 9 and 10 that for flight Mach numbers between 0.8 and 1.0 the trend of thrust ratio with X is opposite to that shown in figure 12. The pressure-rise ratios p_e/p_a corresponding to the thrust ratio values have also been plotted in figure 12, the trend with X being opposite to that of the thrust ratios. The p_e/p_a ratio is a measure of the pumping effect of the ejector.

The results of the calculations to determine the thrust ratios using compressed air as the actuating gas and compare them with values using hot gas are given in figure 13. At the static condition, the thrust ratio with the hot gas is 1.25 compared to 1.31 with the cold gas. The conditions giving the latter value simulate those of the test made in reference 5 as closely as possible with the exception that l/d of 8 was used in the tests and l/d of 10 in the theoretical calculations because the latter value is recommended for perfect mixing which is postulated in the theory. Any difference in results due to friction through the small difference in l/d should be negligible. The test in reference 5 gave a thrust ratio of 1.28 compared to the theoretical value of 1.31. Comparison of an experimental value in reference 5 and a value calculated from the theory for the same conditions as those for the previous comparison between theoretical and experimental values except X was 64 instead of 10 showed a difference of about 6 percent (1.52 experimental; 1.43 theoretical). From the brief comparisons given it is quite probable that the theory will give results that will be fairly accurate in the range of area ratios that are practical for ejectors for jet-propulsion engines. As the flight Mach number increased, the results in figure 13 show that the cold gas gives better thrust increases up to M_o of 0.17 after which the hot gas gives superior results. The results of figure 13 illustrate the importance of using hot actuating gas in ejector experiments. The high thrust values obtained in laboratories can be attributed in no small measure to the use of compressed air for actuating gas and in great measure to very high values of X.

The assumption of uniform pressure, equal to p_a at the mixing-tube inlet was used in the results of figures 9 through 13. The effect of nonuniformity of pressure at the inlet p_a/p_j not equal to 1.0 on thrust ratio over a range of ratio of pressure of air to pressure of gas at the inlet is shown in figure 14. The two curves shown, one for low flight Mach number and the other for a high Mach number, were both calculated for the temperature-ratio condition T_{us}/T_o equal to 9.0. At the low flight Mach number the variation of thrust ratio was negligible over a wide range of pressure ratios greater and less than 1.0 (p_a/p_j from about 0.6 to 1.4). For the same pressure-ratio range at the high flight Mach number the thrust ratio decreased about 17 percent. The range of p_a/p_j was chosen to exceed the range of practical interest. Consequently, it is thought that the trends of thrust ratio with variation of the fundamental parameters obtained with p_a/p_j of 1.0 will be affected little by practical changes of p_a/p_j.

Nozzle at exit of mixing tube ($p_d/p_o = 1.0$).- The thrust augmentation obtained with ejectors with a nozzle at the mixing-tube exit for a flight Mach number M_o of 0.8 and for the extreme values of area ratio X, pressure-rise ratio r, and temperature ratio T_{us}/T_o used in figures 9 and 10 is given in figure 15 over a range of p_a/p_o values. In all calculations the p_a/p_j and p_d/p_o ratios were 1.0. The reasons for the type of curves obtained, four branches with some conditions setting up a range of p_a/p_o values for which no solutions were obtained, has been discussed in the section on selection of conditions. For the conditions of figure 15, the use of an ejector with a mixing-tube exit nozzle provided no thrust increase over the entire range of p_a/p_o values for the solutions that are considered physically significant.

The thrust augmentation with an exit nozzle at a flight Mach number of 2.0 and for conditions that are considered average for turbojet engines with ejectors are shown in figure 16. Again no thrust increase was obtained when the ejector was added for the solutions considered physically significant. Inasmuch as it has been shown that the ejector was more beneficial at low pressure-rise ratios r than at high ratios it was thought that decreasing r from 6, the value used in figure 16, to 1.5 might result in some gain in thrust, all other conditions of figure 16 remaining the same. Consequently, a calculation of the thrust ratio for these conditions at a p_a/p_o value of 7 using a mixing-tube nozzle on the ejector was made. The thrust ratio was 0.98 compared to 0.83 for the same p_a/p_o value shown in figure 16. The thrust increased appreciably but still was less than that for the case of no ejector being used.

Mixing tube discharging to atmosphere $(p_e/p_o > 1.0)$.- In the general treatment of the flow in the ejector it was noted that with subsonic flight speeds p_e/p_o could be greater or less than 1.0 if the flow at the mixing-tube exit or mixing-tube nozzle exit was supersonic and the solution would be physically significant. Also, for a supersonic flight speed, the condition is extended to include subsonic flow at the mixing-tube exit or mixing-tube nozzle exit. Two isolated sets of conditions were chosen to calculate the thrust ratio for the case of an ejector with the mixing tube discharging to atmosphere but with p_e/p_o greater than 1.0, to show the application of the general treatment to this case and to determine the possibilities of thrust augmentation. It is realized that no over-all conclusion as to the benefit deriving from using ejectors for such conditions can be made, but a trend can possibly be noted.

On the basis of a formula derived by Paul R. Hill of the Langley Pilotless Aircraft Research Division for thrust when the exit pressure does not equal the atmospheric pressure, the thrust ratio formula when $p_e \gtrless p_o$ using the mixing tube only was found to be

$$\phi' = (\theta + 1)\frac{M_e\sqrt{\frac{T_e}{T_o}} - M_o}{M_{jF}\sqrt{\frac{T_{jF}}{T_o}} - M_o} + \frac{1}{\gamma}\frac{1}{M_j}\frac{p_o}{p_a}\frac{p_a}{p_j}\sqrt{\frac{T_j}{T_o}}\left[\frac{(X + 1)\left(\frac{p_e}{p_o} - 1\right)}{M_{jF}\sqrt{\frac{T_{jF}}{T_o}} - M_o}\right] \qquad (47)$$

Where ϕ' is the thrust ratio when p_e does not equal p_o and T_j/T_o can be easily determined using equations (27) and (30). When $p_e/p_o = 1.0$ equation (47) reverts to equation (41). For the following sets of conditions,

Condition	M_o	X	T_{us}/T_{os}	p_a/p_o	p_a/p_j	r
43	1.2	1.5	3.0	2.2	1.0	1.5
44	1.2	1.5	3.0	.8	1.0	6.0

it was determined that the thrust ratio was 1.29 for r of 1.5 and 0.88 for r of 6.0. In both cases p_e/p_o was greater than 1.0. The p_a/p_o chosen for r of 1.5 gave a solution which was on the d branch of the curves (subsonic flow at mixing-tube exit and inlet) whereas the other solution was on the b branch (supersonic flow at mixing-tube exit and inlet). The thrust ratio obtained for the one set of conditions is appreciable and shows promising results for the case of using only a mixing tube with no nozzle at its exit. It can be shown, however, that when M_e is supersonic and p_e/p_o is greater or less than 1.0, addition of a nozzle at the mixing-tube exit with p_d/p_o equal to 1.0 will increase the thrust of the installation with ejector without the nozzle.

The fact that from the present theory for the same inlet conditions two physically possible solutions are obtained, one on the a branch and one on the b branch of the curves, has been discussed fully, the causes for such solutions and the determination of actual flow being given. This point especially arises for the case where p_e/p_o does not have to equal 1.0, the flight speed being supersonic.

For frictionless flow and on the basis of the momentum equation it can be shown that the thrust with the ejector can be calculated on the basis of mixing-tube inlet conditions instead of the exit conditions. Inasmuch as the friction term was also based on the secondary-air inlet velocity, the thrust with the ejector can be calculated in the present report on the basis of inlet conditions even with friction. As the same inlet conditions serve to give two sets of conditions at the exit, a and b branches, it is obvious that the thrusts for both sets of conditions are equal. Consequently, from a thrust standpoint, which flow, a or b branch solution, will occur is not important.

Thrust Augmentation of Ram-Jet Units

Mixing tube discharging to atmosphere ($p_a/p_o = 1.0$).- The results of the calculations to determine the thrust augmentation obtained by using ejectors with no nozzle at the mixing-tube exit of ram-jet power units are given in the following table for three supersonic flight Mach numbers at a low and high-area ratio X. In the calculations the pressure at the mixing-tube inlet was assumed to be uniform and equal to p_a ($p_a/p_j = 1.0$).

Area ratio	Flight Mach number	Thrust ratio	Branch of general curves on which solution falls
X	M_o		
1.5	1.4	1.42	c[1]
	2.0	-1.40	a
		.52	b
		1.81	c[1]
	3.0	-2.45	a
		.60	b
		2.07	c[1]
6.0	1.4	-3.45	a
		-.74	b
		1.59	c[1]
	2.0	-6.29	a
		-.13	b
		2.67	c[1]
	3.0	-8.83	a
		-.12	b
		3.50	c[1]

[1]Considered not physically significant

The results in the table show that over the range of conditions given the only solutions showing augmentation of thrust are those for the c branch of the curve which is not considered physically attainable. The c-branch results show values in the range of calculated results of other ejector reports at high flight speeds. Very probably these other results have the same physical limitations as the present results. The discussion given in a previous section where figure 9 results were presented on the flows that will actually be obtained for a p_a/p_o value that results in a solution on the c branch of the curves applies again here. The pressure ratio p_e/p_o for the actual flows would be other than 1.0 and equation (47) would be used to calculate the thrust ratio. The thrust ratios calculated by means of equation (47) for the actual flows, which it is thought should be used in place of the c-branch thrust ratios given in the foregoing table, would possibly show that the ejector aided the thrust. Also, for some other inlet conditions (p_a/p_o value) for which p_e/p_o was not equal to 1.0, which is physically possible for all three branches a, b, and d, of the curves because the flight Mach number is supersonic, might show a gain in thrust through use of an ejector. No calculations of thrust ratio for the conditions discussed here were made because the thrust-augmentation problem is only given to illustrate the application of the compressible flow treatment and the work was purposely limited.

Nozzle at exit of mixing tube ($p_d/p_o = 1.0$).- The thrust augmentation of ram-jet units obtained with a nozzle placed at the exit of the mixing tube of the ejector for flight Mach numbers of 1.4, 2.0, and 3.0 at two values of area ratio, X of 1.5 and 6.0, is given in figure 17. It is apparent from the figure that the thrust augmentation obtained for inlet conditions such that the d branch of the curves is applicable is appreciable. The thrust ratios for the condition of subsonic flow at the inlet and sonic at the exit of the mixing tube (lower limit of d branch) obtained from the figure are given in the following table.

Area ratio	Flight Mach number	Thrust ratio
X	M_o	ϕ
1.5	1.4	1.34
	2.0	1.33
	3.0	1.29
6.0	1.4	1.36
	2.0	1.44
	3.0	1.37

As p_a/p_o increases above the value for which choking occurs at the exit the thrust ratio increases somewhat. At the upper p_a/p_o limit, for instance, at M_o of 1.4 and X of 6.0 the thrust ratio is about 1.7. The results shown give promise of improvement of performance of airplanes or missiles at high speeds by applying ejectors when the mass of fluid flowing through the ram-jet unit without and with the ejector is the same. The practical aspects of providing metals or cooling to withstand the high temperatures downstream of the jet unit that exist for the conditions given are problems not considered herein. Experiments by other investigators with supersonic nozzles have shown that the thrust of jet-propulsion devices is decreased little by cutting off the end of nozzles placed at the exit of the jet unit. This can be shown to be true only for low supersonic Mach numbers at the nozzle exit. Consequently, it is possible that the thrust augmentation with no nozzle at the mixing-tube exit for those cases discussed in the preceding section on ram-jet units for which the nozzle exit Mach number does not greatly exceed a value of 1.0 would be only a little less than the values given in the preceding table.

Comparison of thrust augmentation of small ram jets with ejectors and large ram jets without ejectors of equal external frontal area.- The ratio of the thrust of the large ram jet without an ejector to the thrust of the small ram jet with an ejector, using the same fuel quantity in each case but with the combustion air flow in the former case equal to the sum of the primary and secondary air flow in the latter case, are given in the following table. For these conditions with all air taken in at the front of the installations and for the same altitude in both cases, the frontal areas of the setups have been assumed equal. The table in effect gives the ratio of thrusts obtained by either enlarging the jet area or increasing the area the same amount by adding an ejector.

Area ratio	Flight Mach number	Thrust ratio
1.5	1.4	1.80
	2.0	1.70
	3.0	1.56
6.0	1.4	2.12
	2.0	2.01
	3.0	1.82

From these results it is evident that the thrust of a small ram jet can be increased more by enlarging it than by adding an ejector with a nozzle at the mixing-tube exit. The ram jet with the ejector has the advantage of the cooling action of the secondary air which is important at the high temperatures encountered. An advantage of the large ram-jet is that because of its low fuel-air ratio the dissociation losses are low and, consequently, the heat input as well as the efficiency is higher than for the small ram jet. If it had been postulated that the secondary air for the ejector would be taken in at the rear of the installation so that the frontal area of the small ram-jet installation would be less than that of the large ram-jet installation, it is thought that the thrust increase due to smaller installation size would not have overcome the large increase of thrust when the large ram jet is used.

GENERAL DISCUSSION

The foregoing sections have given the general treatment of compressible flow in an ejector and an application of it to a particular problem. The latter work was not complete as it was not the object to explore the possible application of ejectors to jet-propulsion units thoroughly. The present section has been written with the object of giving a brief outline of the methods of application of the theory to some other problems, a summary picture of the possibilities of the ejector as a thrust device for jet-propulsion units, and some future work needed.

Outline of Methods of Applying the General Theory to Some Other Problems

Pumping.- The ejector as a cooling augmentor operates as a pump to cause air to flow from a region of low pressure to one of high pressure at the ejector exit. The region of low pressure can be a space behind an air-cooled internal-combustion engine or the exit of a duct passing around a jet-propulsion engine, the air passing through the duct cooling the surface of the metal enclosure around the engine. As a pump the ejector shows promise, but the practical aspects of the problems, such as structural failure due to vibration, are difficult to overcome. The ejector has been explored more completely from the standpoint of cooling augmentation of internal-combustion engines than for any other purpose. A theory for the latter purpose and some experimental results have been given by Manganiello and Bogatsky in reference 12. In the present section, the general theory has been applied more specifically to the problem of jet engine cooling where compressible effects are more liable to occur. The theory presented below can be applied, however, to internal-combustion engine cooling.

From cooling theory it can be shown that when an ejector is used on an engine the ejector parameters H_a/p_o, H_u/p_o, T_{us}/T_{as}, and θ are fixed for given conditions of altitude, flight speed, engine power, fuel-air ratio, and engine or metal surface temperatures. The problem resolves itself into determining the area ratio X and p_a/p_o for the parameters H_a/p_o, H_u/p_o, and T_{us}/T_{as} such that the desired ratio of air masses θ is obtained. The total pressure H_u and stagnation temperatures T_{us} are the conditions of the exhaust gas downstream of the engine. From the general theory it can be shown that with a pressure H_a at the inlet and if p_a/p_j is assumed equal to 1.0, the cooling design problem is solved by plotting p_e/p_o and θ versus p_a/p_o for ranges of the latter from 0 to H_a/p_o for the conditions given and for several values of X using equations (26), (29), (31), (32), (33), (36), (37), and (38) to determine p_e/p_o and θ. In the equations the p_a/p_j, $1 + \frac{\gamma - 1}{2} M_o^2$, T_{us}/T_{os}, and r terms are replaced with 1.0, $(H_a/p_o)^{\frac{\gamma-1}{\gamma}}$, T_{us}/T_{as}, and H_u/H_a respectively, wherever they occur. The ejector design X is chosen from the sets of curves for each X such that θ is obtained for a physically significant solution.

<u>Thrust augmentation of internal-combustion engines</u>.- The problem of attaching ejectors to exhaust pipes of internal-combustion engines to augment the thrust of the propeller-driven airplane has been considered from time to time. Some thrust increase has been obtained in some cases (references 4 and 13). If intermittent jet effects are disregarded, the determination of the flow conditions that will exist in the ejector can be accomplished by using the general treatment in exactly the same manner as that used in the problem given as an example in the present report. The thrust-ratio equations will be somewhat different, however, from those given for that problem.

Future Explorations of Ejector Problems

For the conditions, studies and neglecting duct losses up to the ejector and extra outside losses due to the ejector, the present calculations showed thrust augmentation of jet-propulsion units for some isolated cases. The extreme limits of thrust ratio obtained for each set of conditions listed in table I or in the special table for which p_e/p_o was not 1.0, are given in table II. In general, thrust increases were obtained at take-off when the ejector was used and for high subsonic flight Mach numbers when the pressure-rise ratio was low and heat input high. The effect of increase of the area ratio X at low Mach numbers when thrust augmentation was obtained was to increase the thrust with the ejector, but, in general, the reverse was true at high speeds when the mixing tube only was used. Except for take-off thrust augmentation, the only thrust augmentation that gave promise of increasing the performance of jet units appreciably was that obtained for ram-jet units at supersonic flight speeds using a nozzle at the mixing-tube exit. The effect of an increase of X for the latter cases was to increase the thrust, which effect is opposite to that given for some conditions with mixing tube alone at high subsonic speeds. It was also shown, however, that a large ram jet using no more fuel than the small ram jet with the ejector would give more thrust than the latter power plant. Where cooling or boundary-layer control is required in addition to thrust increase, however, the use of ejectors with small ram jets is desirable.

Although the addition of an ejector did not show great promise of performance increase of jet-propulsion units from the limited study made herein, further work should be done to explore the performance to be obtained for greater ranges of conditions including those for which physical solutions are possible when the pressure at the exit of the mixing tube p_e and at the exit of the mixing-tube nozzle p_d do not equal the atmospheric pressure. There were indications as pointed out in the report that such conditions may be beneficial to performance. The use of a method of analysis whereby the velocity of the airplane is constant with and without the ejector, which postulates a change of airplane size because the thrust changes, and a change in the size of the jet nozzle when the ejector is used to obtain the same mass of gas as when the ejector is not used, may not be suited to all purposes. A method of analysis other than that given would be needed for each specific problem.

Inasmuch as differences between results calculated from the present theory and experimental results were generally much less than those given heretofore in the literature, the application of the theory to other problems can be made with greater confidence. The pumping problem should be explored, especially with regard to ranges of conditions as exist for cooling of jet-propulsion installations. An advantage of use of the ejector is the possibility of noise reduction to be obtained by its uses which is especially important on commercial airplanes with the rapid growth of airports around communities that is expected.

Fundamental work of a theoretical and experimental nature on the mixing of gases should be made which will have application to all problems of ejector application and to other flow fields. Some work exists on the mixing theory (references 1, 2, and 14) which should be followed up to obtain a more complete theory. The existence or nonexistence of flow conditions which were questionable in the foregoing analysis, such as various combinations of supersonic Mach numbers at the mixing-tube exit and supersonic and subsonic Mach numbers of the primary gas and secondary air at the mixing-tube inlet, should be determined, as the performance to be obtained for some flows is appreciable. The work should be extended to include multiejectors as well as single ejectors because most experiments show the former to be superior with respect to thrust augmentation. The efficiencies of the systems as well as the thrust augmentation should be included in order to obtain the over-all picture of the use of ejectors. The experimental work should be done with heated augmentation air and for conditions other than static in case good thrust increases are indicated by further exploration. The practical problems involved because of vibration and high temperatures will also require much experimental work.

The launching of such programs as outlined depends on further exploration of the ejector possibilities for which the present general treatment was derived as the first step in such exploration and as the primary object of this report.

CONCLUDING REMARKS

1. A comprehensive treatment of compressible flow in single ejectors with straight circular mixing tubes has been developed which has led to a method with certain limitations which permits calculation of physical conditions in ejectors of varying geometries for any set of flow actuating conditions.

2. Certain assumptions, for example, the perfect mixing of the fluids in the mixing tube are of questionable validity. It is thought, however, based in part on calculations made, that for practical purposes satisfactory agreement between calculated and experimental conditions in an ejector using the present theory will be obtained.

3. The general theory can be applied to any problem concerned with pumping or thrust augmentation to be obtained through use of ejectors, as all such problems involve knowledge of physical conditions in the ejector.

4. From the example used to show the application of the general theory to the problem of thrust augmentation of jet-propulsion engines with addition of single ejectors whose geometry was varied to obtain ideal conditions through the flight range, it was determined over a limited range of exploratory conditions that generally at take-off but otherwise only in isolated instances for certain combinations of flight and engine conditions was the thrust increased. The thrust was often decreased rather than increased when the ejector was added.

5. If an ejector with a nozzle at the mixing-tube exit is added to a ram-jet installation operating at supersonic flight speeds for the direct purpose of using the secondary air for cooling or boundary-layer control, good thrust augmentation is obtained as an indirect result. For the direct purpose of thrust augmentation, however, more thrust is obtained with a large ram-jet installation without an ejector whose mass flow rate is equal to the sum of the primary and secondary flow rates in the ejector setup.

6. In the calculations it was necessary to rule out certain solutions that were considered physically impossible and for which very optimistic thrust-augmentation results were obtained. Previously reported optimistic calculated results of other investigators very probably are based on such physically impossible solutions.

7. Although the calculations in the example showed only isolated instances of thrust augmentation of jet engines through use of ejectors, no conclusion as regards their use for this purpose can be made until the problem is explored for greater ranges of conditions than those used in this report which were limited because the object of the example was only to show the application of the theory.

8. On the basis of the present work and work of others on pumping, the ejector shows some promise so that more exploratory work using the present theory to determine performance results for other problems and fundamental experimental research to determine mixing processes are warranted.

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National Advisory Committee for Aeronautics
Langley Field, Va.

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TABLE I

CONDITIONS USED IN THRUST AUGMENTATION CALCULATIONS

<table>
<tr><th>Condition</th><th>Type of unit</th><th>l/d</th><th>X</th><th>M_o</th><th>r</th><th>T_{us}/T_o</th><th>p_a/p_j</th><th>Ejector construction</th><th>Air-fuel ratio</th></tr>
<tr><td>1</td><td rowspan="22">Turbo-jet</td><td rowspan="42">10</td><td rowspan="12">1.5</td><td rowspan="20">0-1.4</td><td>1.5</td><td rowspan="4">3</td><td rowspan="21">1.0</td><td rowspan="25">Straight circular mixing tube</td><td rowspan="30"></td></tr>
<tr><td>2</td><td>3.0</td></tr>
<tr><td>3</td><td>6.0</td></tr>
<tr><td>4</td><td>12.0</td></tr>
<tr><td>5</td><td>1.5</td><td rowspan="4">6</td></tr>
<tr><td>6</td><td>3.0</td></tr>
<tr><td>7</td><td>6.0</td></tr>
<tr><td>8</td><td>12.0</td></tr>
<tr><td>9</td><td>1.5</td><td rowspan="4">9</td></tr>
<tr><td>10</td><td>3.0</td></tr>
<tr><td>11</td><td>6.0</td></tr>
<tr><td>12</td><td>12.0</td></tr>
<tr><td>13</td><td>3.0</td><td rowspan="4">1.5</td><td rowspan="2">3</td></tr>
<tr><td>14</td><td>6.0</td></tr>
<tr><td>15</td><td>3.0</td><td rowspan="2">9</td></tr>
<tr><td>16</td><td>6.0</td></tr>
<tr><td>17</td><td>3.0</td><td rowspan="4">12.0</td><td rowspan="2">3</td></tr>
<tr><td>18</td><td>6.0</td></tr>
<tr><td>19</td><td>3.0</td><td rowspan="2">9</td></tr>
<tr><td>20</td><td>6.0</td></tr>
<tr><td>21</td><td>10.0</td><td>0</td><td>1.5</td><td>3</td></tr>
<tr><td>22</td><td>6.0</td><td>0.4 and 1.4</td><td>12.0</td><td>9</td><td>0.2-1.4</td></tr>
<tr><td>23</td><td rowspan="3">Laboratory test simulation</td><td>10.0</td><td rowspan="2">0-0.8</td><td rowspan="3">$\frac{3530}{H_o}$ *</td><td>1.038</td><td rowspan="20">1.0</td></tr>
<tr><td>24</td><td>10.0</td><td>4.63</td></tr>
<tr><td>25</td><td>64.0</td><td>0</td><td>1.038</td></tr>
<tr><td>26</td><td rowspan="5">Turbo-jet</td><td rowspan="2">1.5</td><td rowspan="4">0.8</td><td>1.5</td><td>3</td><td rowspan="5">Nozzle at mixing-tube exit</td></tr>
<tr><td>27</td><td>12.0</td><td>9</td></tr>
<tr><td>28</td><td rowspan="3">6.0</td><td>1.5</td><td>3</td></tr>
<tr><td>29</td><td>12.0</td><td>9</td></tr>
<tr><td>30</td><td>2.0</td><td>6.0</td><td>3</td></tr>
<tr><td>31</td><td rowspan="12">Ram jet</td><td rowspan="3">1.5</td><td>1.4</td><td rowspan="12">1.0</td><td rowspan="12">$\frac{T_{os}+\Delta T}{T_o}$</td><td rowspan="6">Straight circular mixing tube</td><td rowspan="12">15</td></tr>
<tr><td>32</td><td>2.0</td></tr>
<tr><td>33</td><td>3.0</td></tr>
<tr><td>34</td><td rowspan="3">6.0</td><td>1.4</td></tr>
<tr><td>35</td><td>2.0</td></tr>
<tr><td>36</td><td>3.0</td></tr>
<tr><td>37</td><td rowspan="3">1.5</td><td>1.4</td><td rowspan="6">Nozzle at mixing-tube exit</td></tr>
<tr><td>38</td><td>2.0</td></tr>
<tr><td>39</td><td>3.0</td></tr>
<tr><td>40</td><td rowspan="3">6.0</td><td>1.4</td></tr>
<tr><td>41</td><td>2.0</td></tr>
<tr><td>42</td><td>3.0</td></tr>
</table>

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* $H_o = 2116\left[1+0.2M_o^2\right]^{3.5}$

TABLE II

RANGE OF PHYSICALLY POSSIBLE THRUST RATIOS

OBTAINED FOR EACH SET OF CONDITIONS USED

Condition	Range of thrust ratios, ϕ	
	p_e/p_o or p_d/p_o = 1.0	$p_e/p_o > 1.0$
1	-2.02 to 1.07	
2	-1.32 to 1.08	
3	-.99 to .93	
4	-.52 to .94	
5	-.82 to 1.05	
6	-.59 to 1.05	
7	-.46 to .88	
8	-.38 to .98	
9	-.27 to 1.08	
10	-.27 to 1.05	
11	-.22 to .81	
12	-.190 to 1.00	
13	-3.52 to 1.14	
14	-6.54 to 1.22	
15	-1.00 to 1.13	
16	-2.79 to 1.20	
17	-1.22 to .92	
18	-2.04 to 1.21	
19	-.39 to .77	
20	-.89 to 1.10	
21	1.28	
22	M_o, 0.4 .91 to .98	
	M_o, 1.4 .56 to .74	
23	-9.00 to 1.31	
24	.35 to 1.25	
25	1.43	
26	-.90 to .96	
27	-.26 to .98	
28	-4.85 to .89	
29	-.68 to .98	
30	-3.66 to .85	
31	Solution on c branch	
32	-1.40 to .52	
33	-2.45 to .60	
34	-3.45 to -.74	
35	-6.29 to -.13	
36	-8.83 to -.12	
37	-2.10 to 1.51	
38	-1.40 to 1.43	
39	-.65 to 1.33	
40	-8.60 to 1.76	
41	-2.41 to 1.64	
42	-3.88 to 1.47	
43		1.29
44		.88

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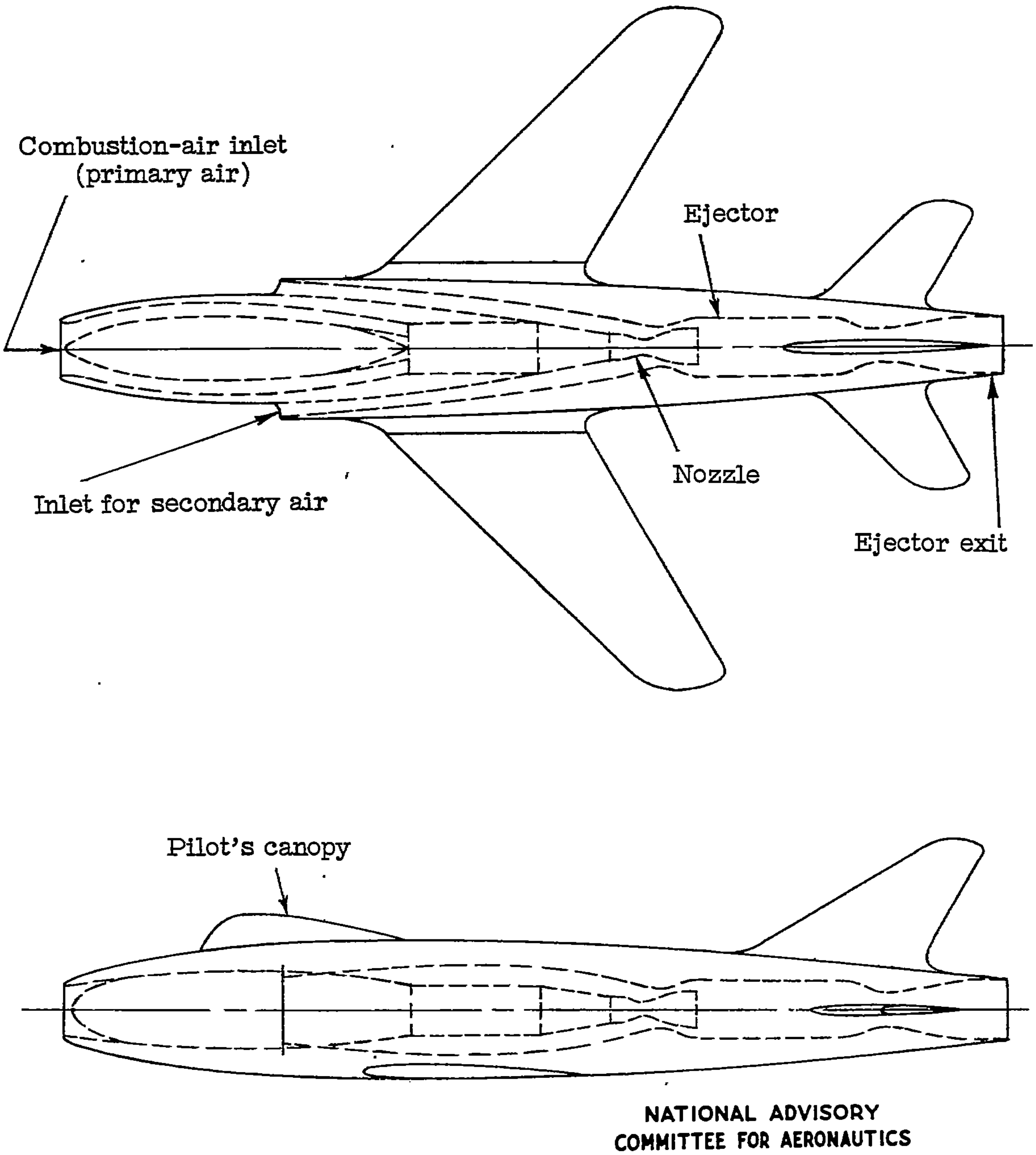


Figure 1.- Jet-propelled airplane illustrating installation of ejector.

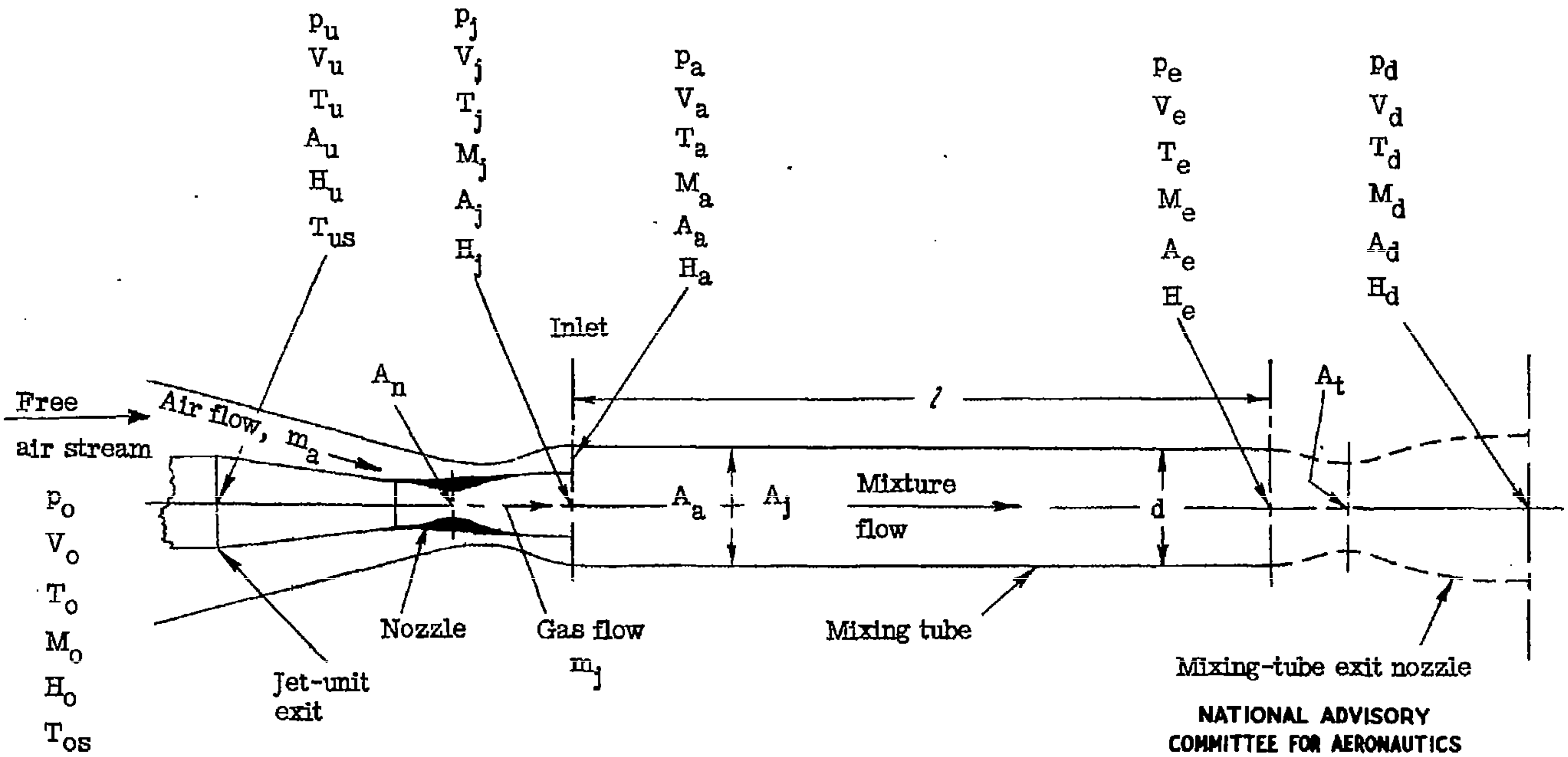


Figure 2.- Ejector-jet system illustrating nomenclature.

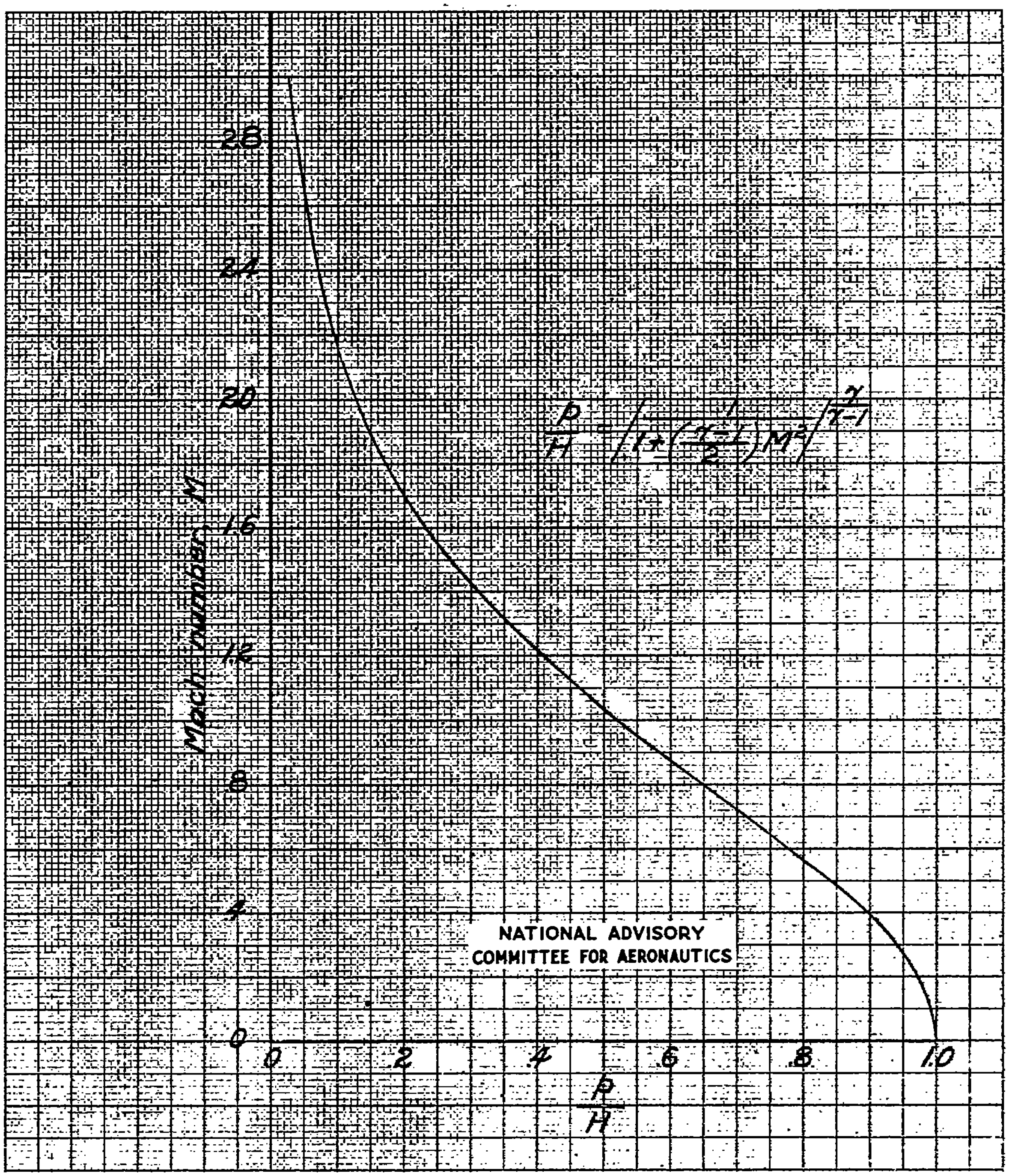


Figure 3.- Variation of Mach number with ratio of static pressure to stagnation pressure.

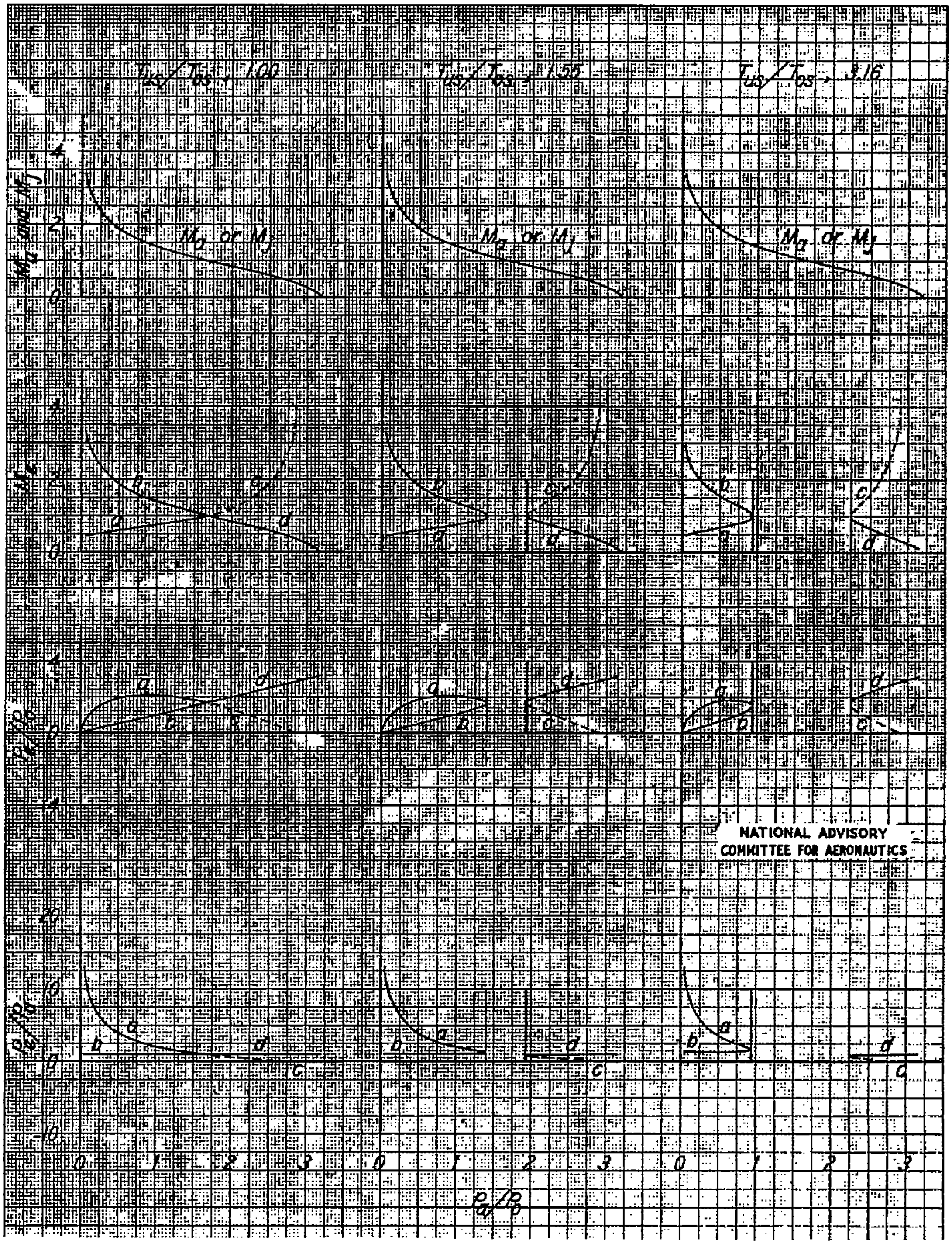


Figure 4.- Effect of heating on conditions in mixing tube (r, 1.0; X, 1.0; M_o, 1.4 or H_o/p_o, 3.18; without friction).

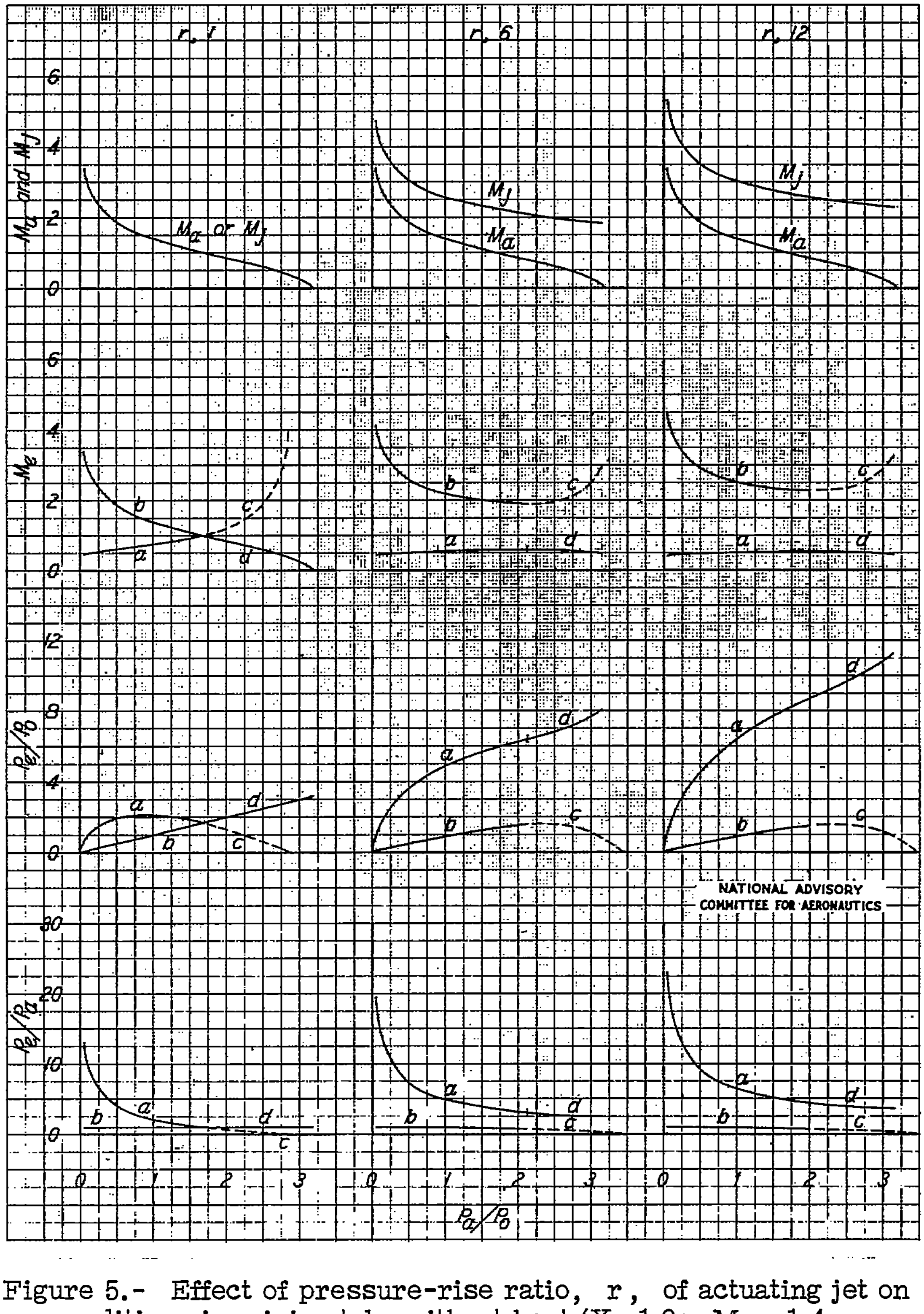


Figure 5.- Effect of pressure-rise ratio, r, of actuating jet on conditions in mixing tube without heat (X, 1.0; M_o, 1.4 or H_o/p_o, 3.18; T_{us}/T_{os}, 1.0; without friction).

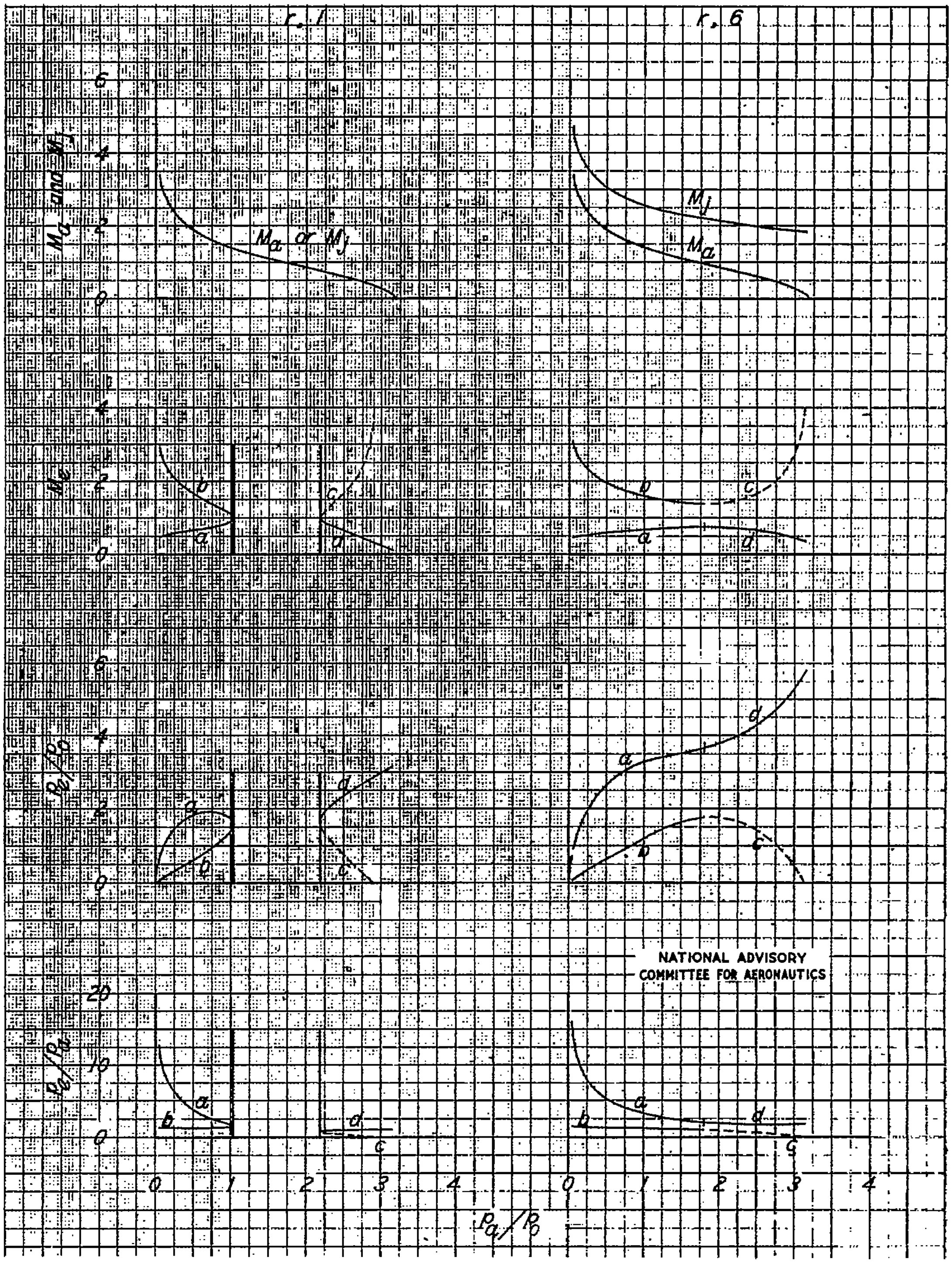


Figure 6.- Effect of pressure-rise ratio, r, of actuating jet on conditions in mixing tube with heat (X, 3; M_o, 1.4 or H_o/p_o, 3.18; T_{us}/T_{os}, 3.16; without friction).

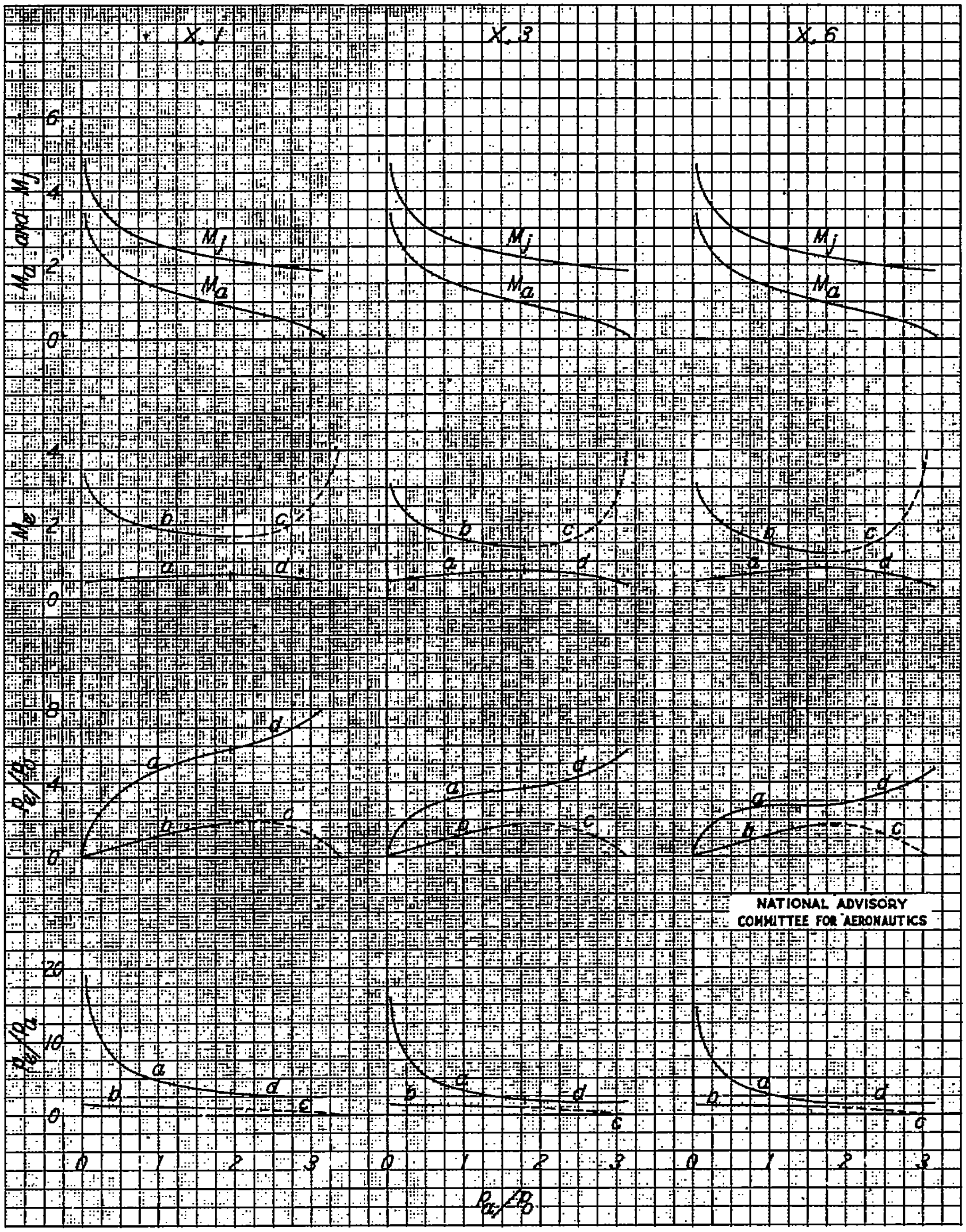


Figure 7.- Effect of ratio of secondary-air area at mixing-tube inlet to area of jet nozzle exit, X, on conditions in mixing tube (r, 6.0; M_o, 1.4 or H_o/p_o, 3.18; T_{us}/T_{os}, 3.16; without friction).

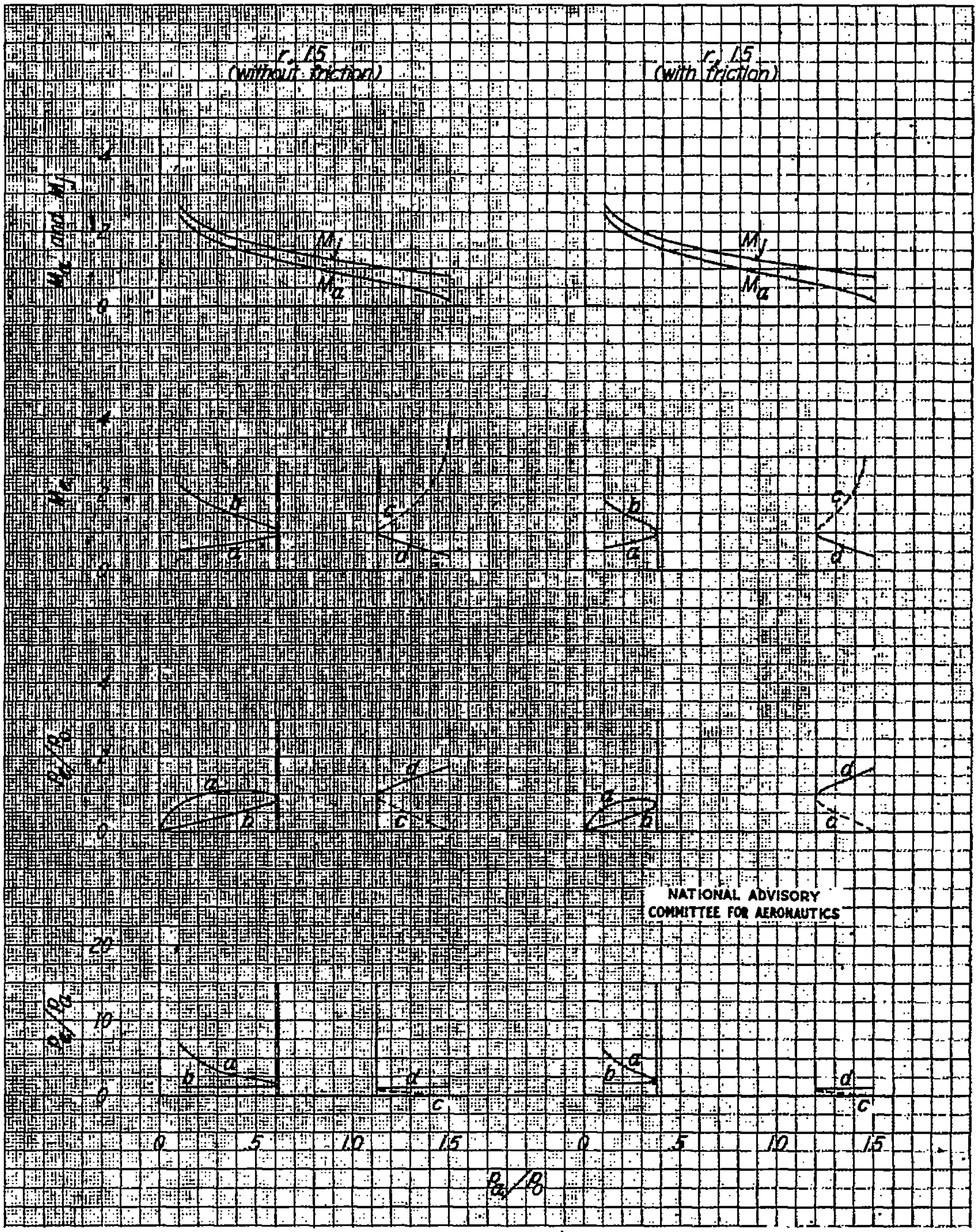
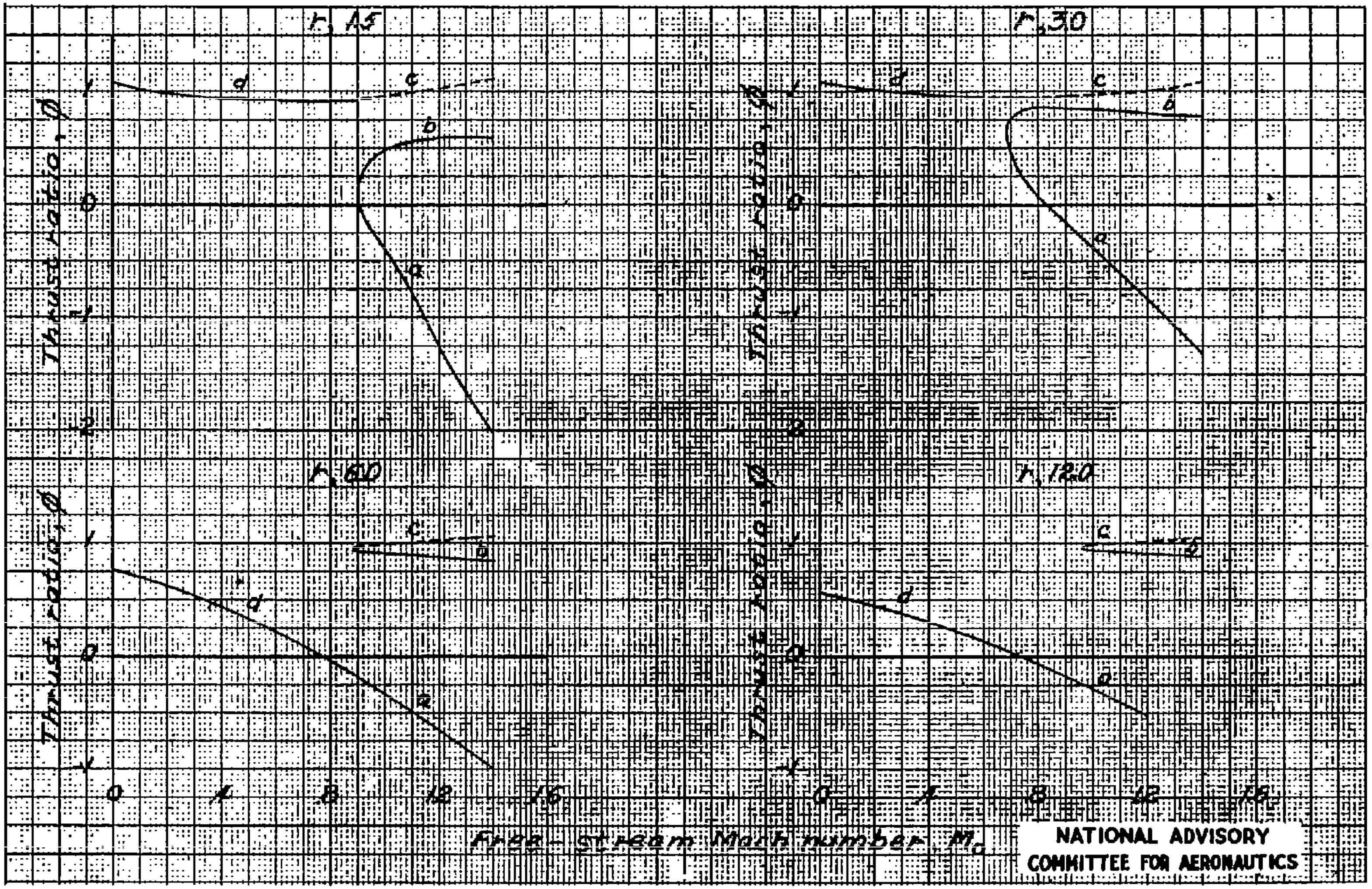
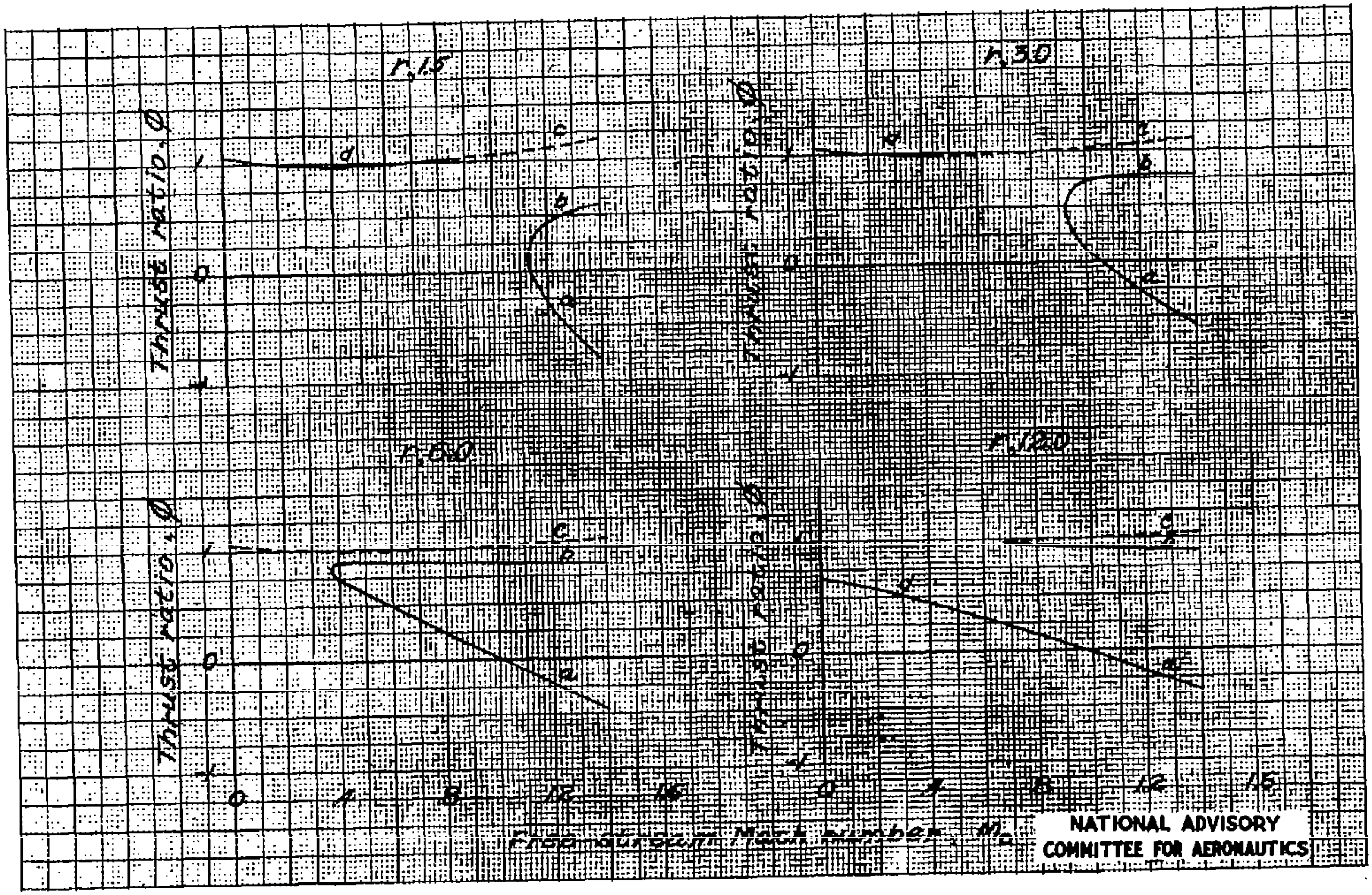


Figure 8.- Effect of friction on conditions in mixing tube (X, 1.5; M_o, 0.8 or H_o/p_o, 1.54; T_{us}/T_{os}, 2.66).



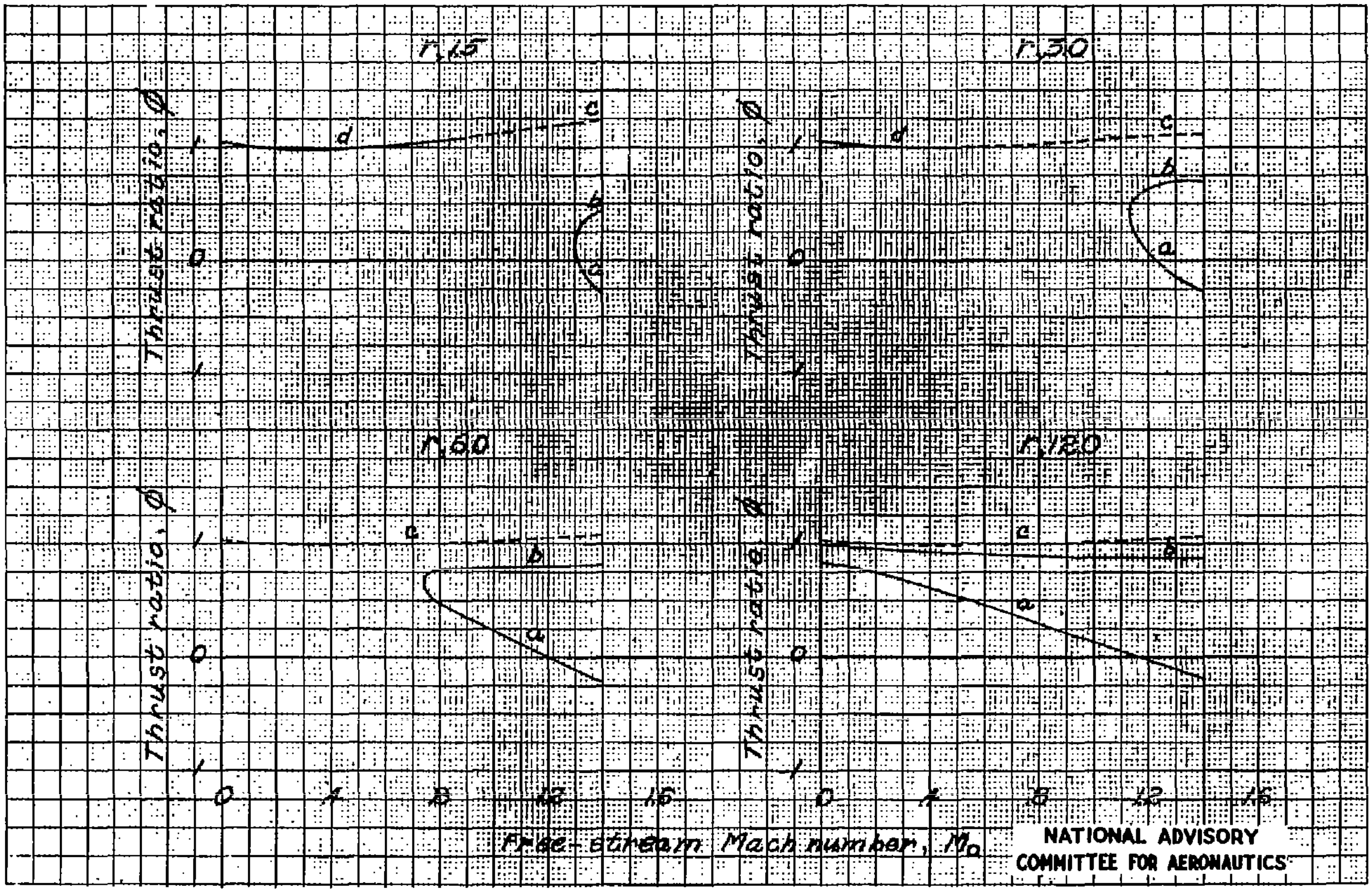
(a) T_{us}/T_o, 3.0.

Figure 9.- Effect of heat, T_{us}/T_o, and pressure-rise ratio, r, of actuating jet on thrust augmentation over a range of free-stream Mach numbers (X, 1.5; with friction; p_e/p_o, 1.0).



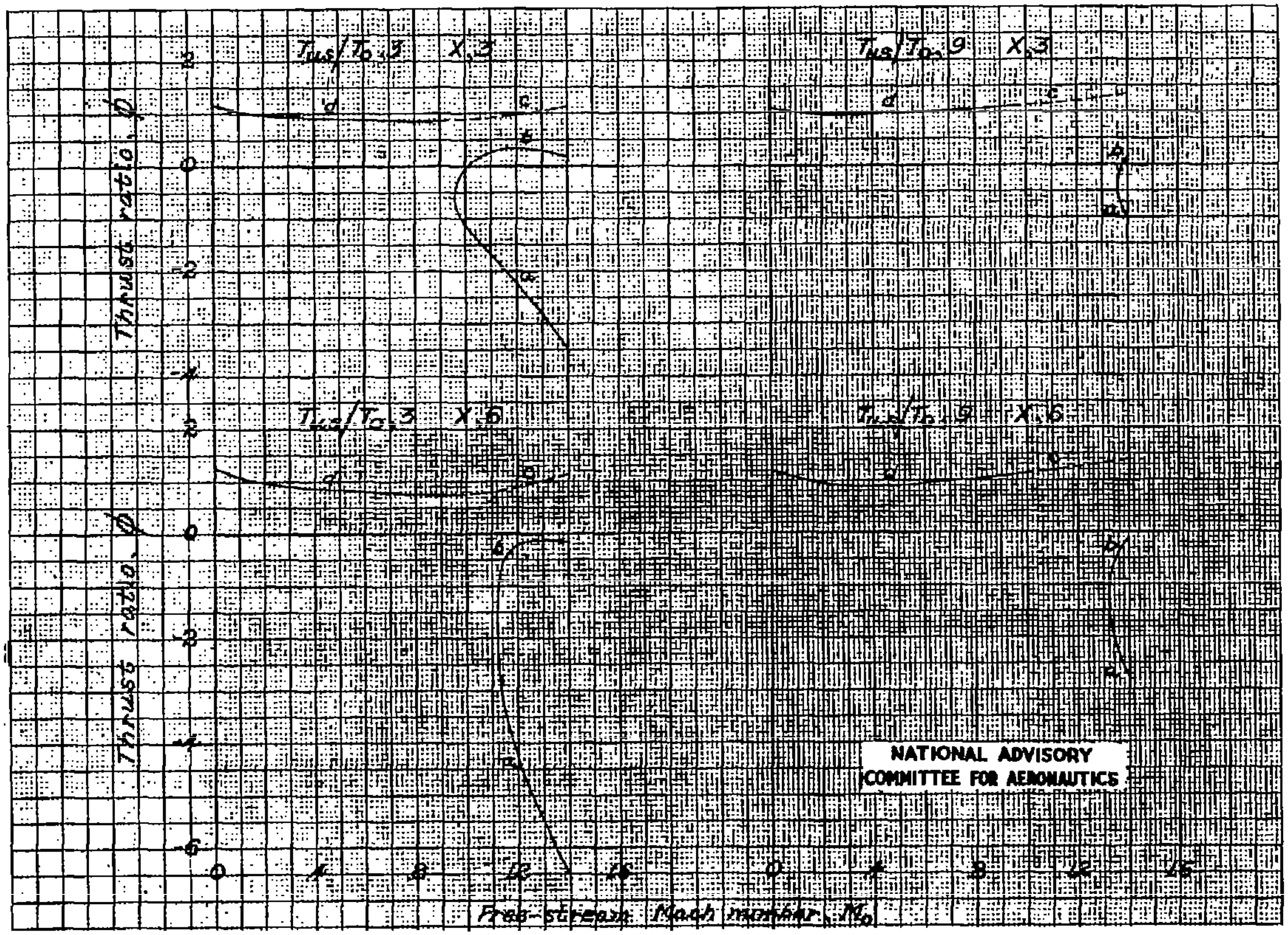
(b) T_{us}/T_o, 6.0.

Figure 9.- Continued.



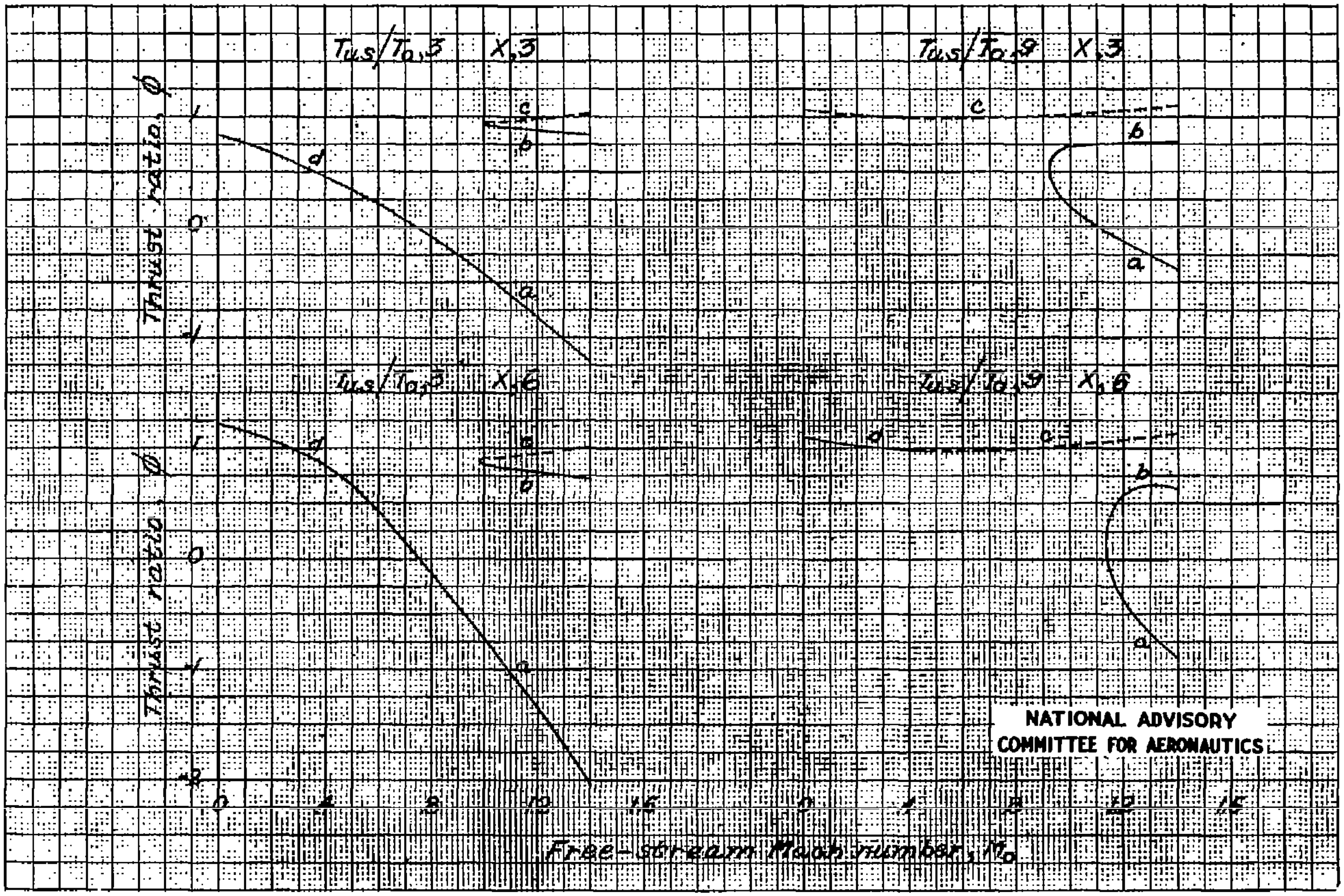
(c) T_{us}/T_o, 9.0.

Figure 9.- Concluded.



(a) Pressure-rise ratio, r, 1.5.

Figure 10.- Effect of heat, T_{us}/T_0, and pressure-rise ratio, r, of actuating jet on thrust augmentation over a range of free-stream Mach numbers (X, 3.0 and 6.0; with friction; p_e/p_0, 1.0).



(b) Pressure-rise ratio, r, 12.0.

Figure 10.- Concluded.

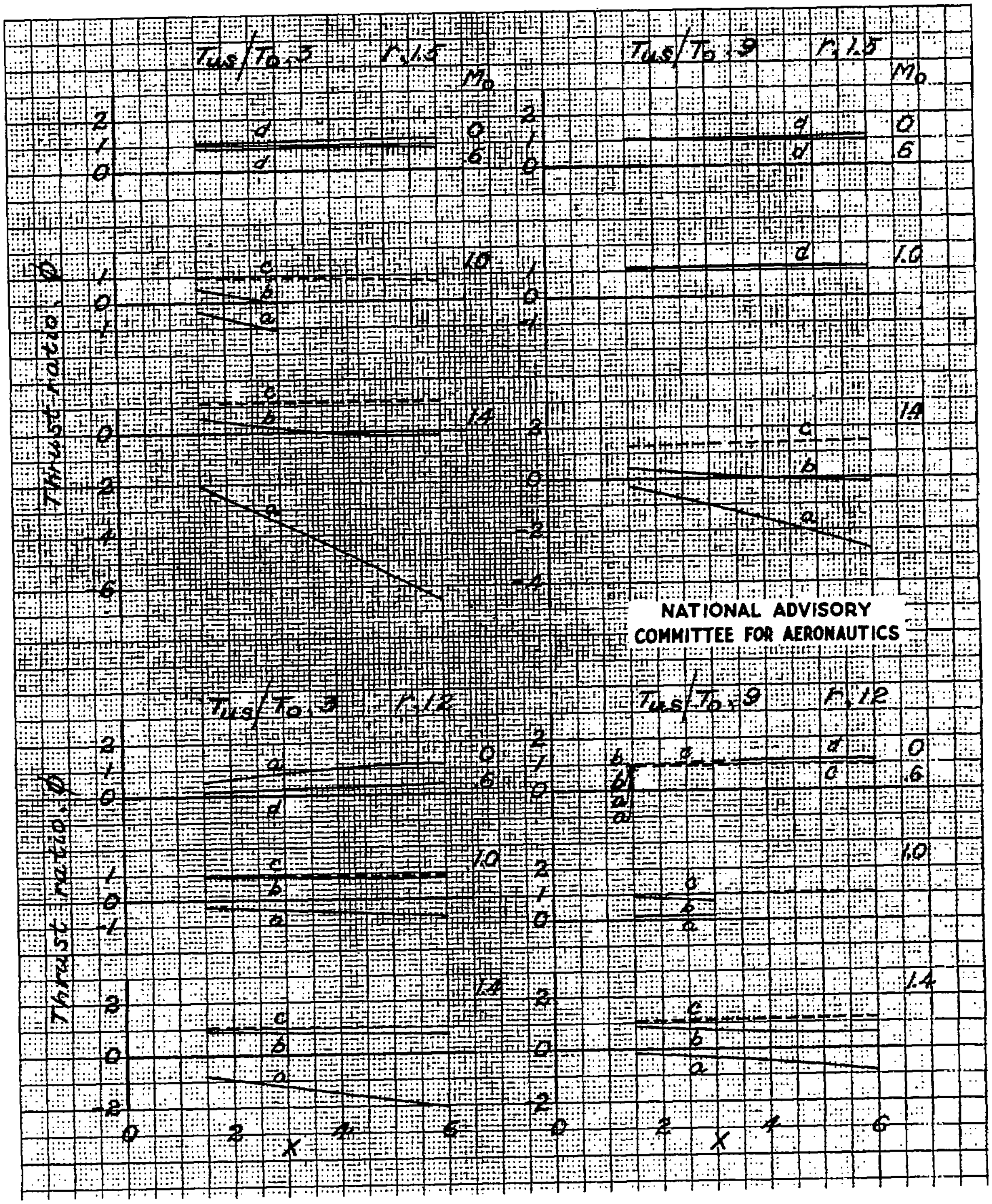


Figure 11.- Effect of area ratio, X, on thrust augmentation for several free-stream Mach numbers.

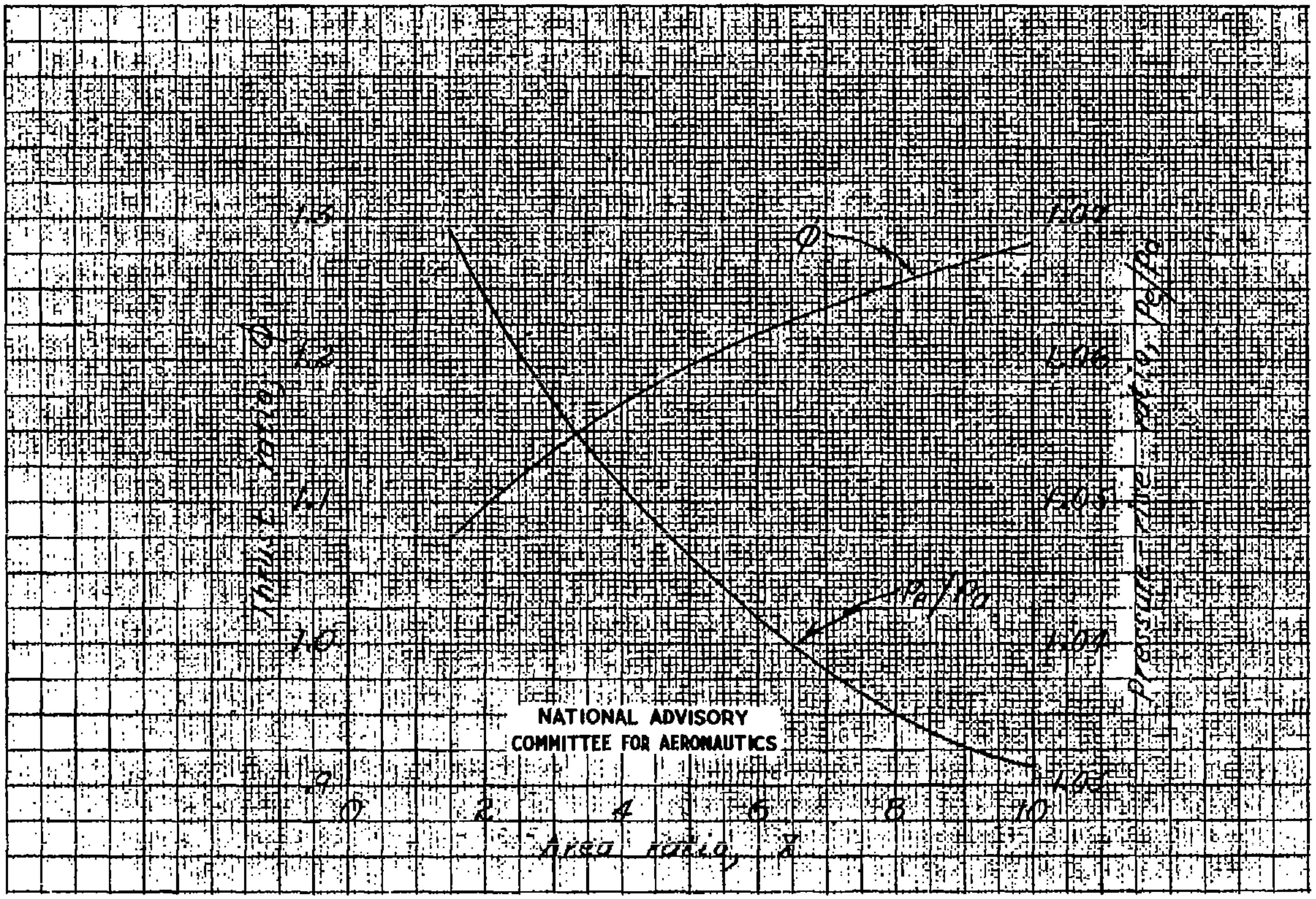


Figure 12.- Effect of area ratio, X, on thrust ratio and pressure-rise ratio in mixing tube (M_o, 0; r, 15; T_{us}/T_o, 3.0).

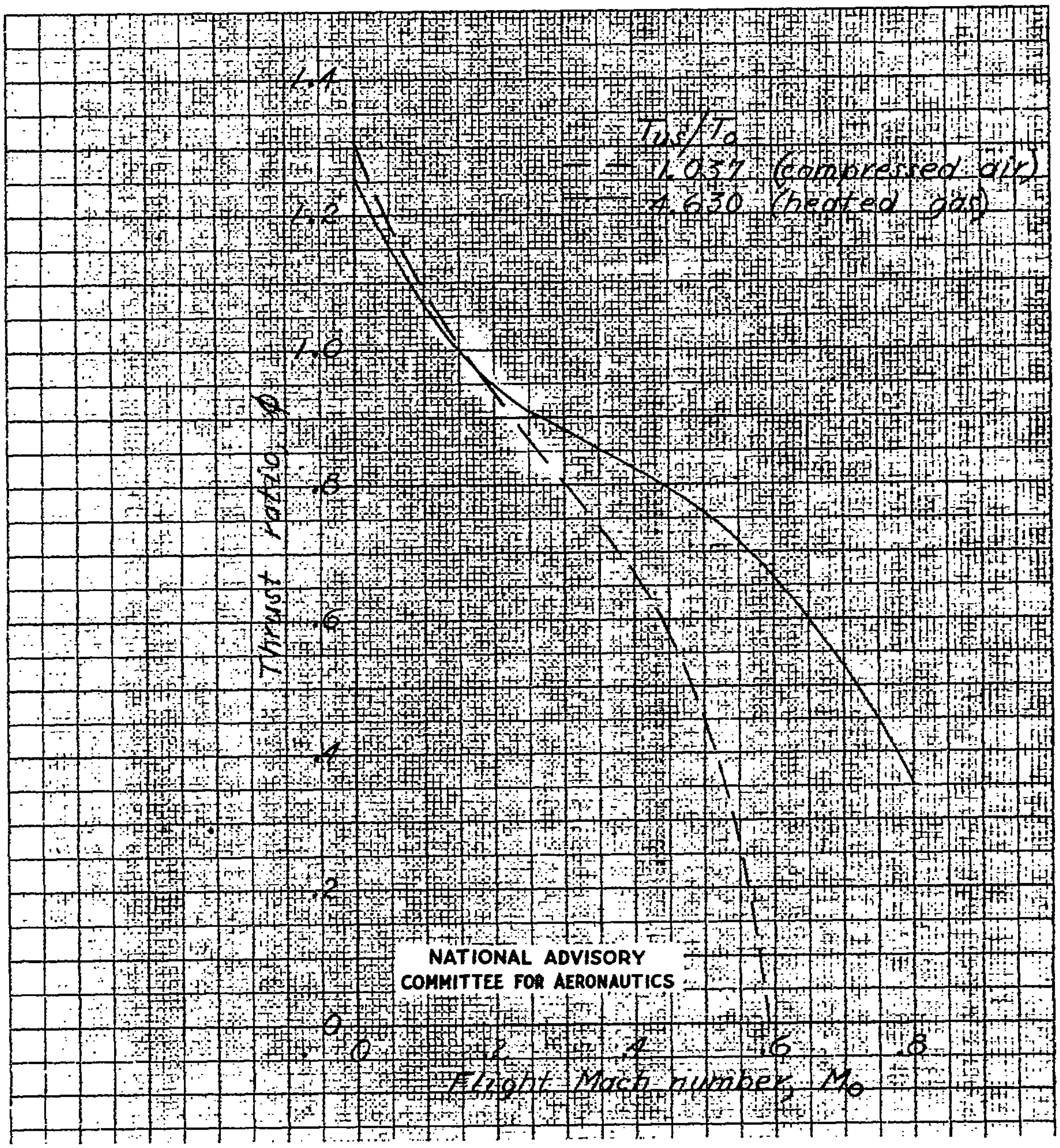


Figure 13.- Effect of using compressed air and heated gas as the actuating fluid on thrust ratios over a range of flight Mach numbers (X, 10; r, 3530/H_o; p_o, 2116 lb/sq ft; l/d, 10).

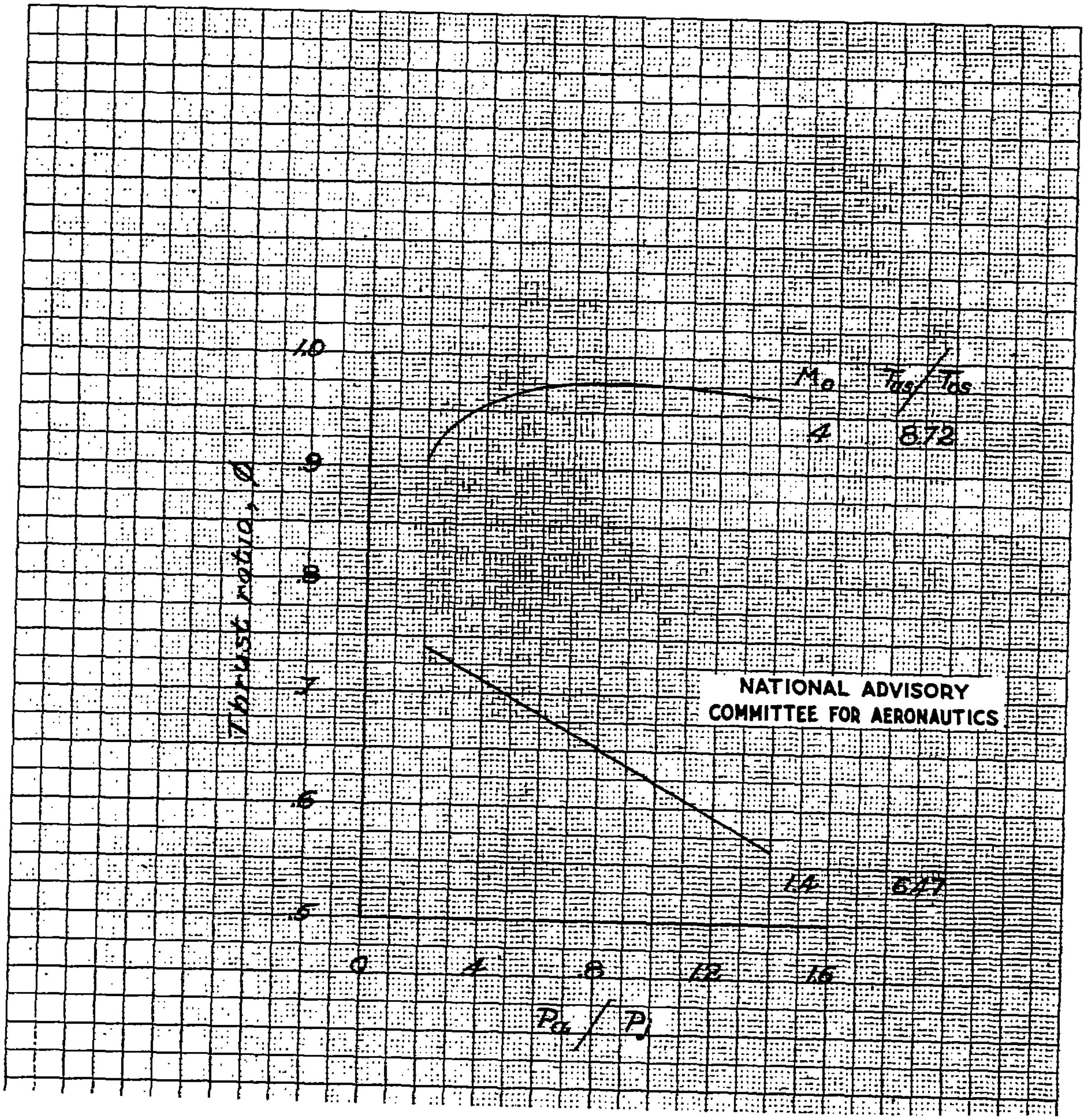


Figure 14.- Effect of ratio of pressures of secondary air and primary air at mixing-tube inlet, p_a/p_j, on thrust ratio (r, 12.0; with friction; p_e/p_o, 1.0, X, 6.0).

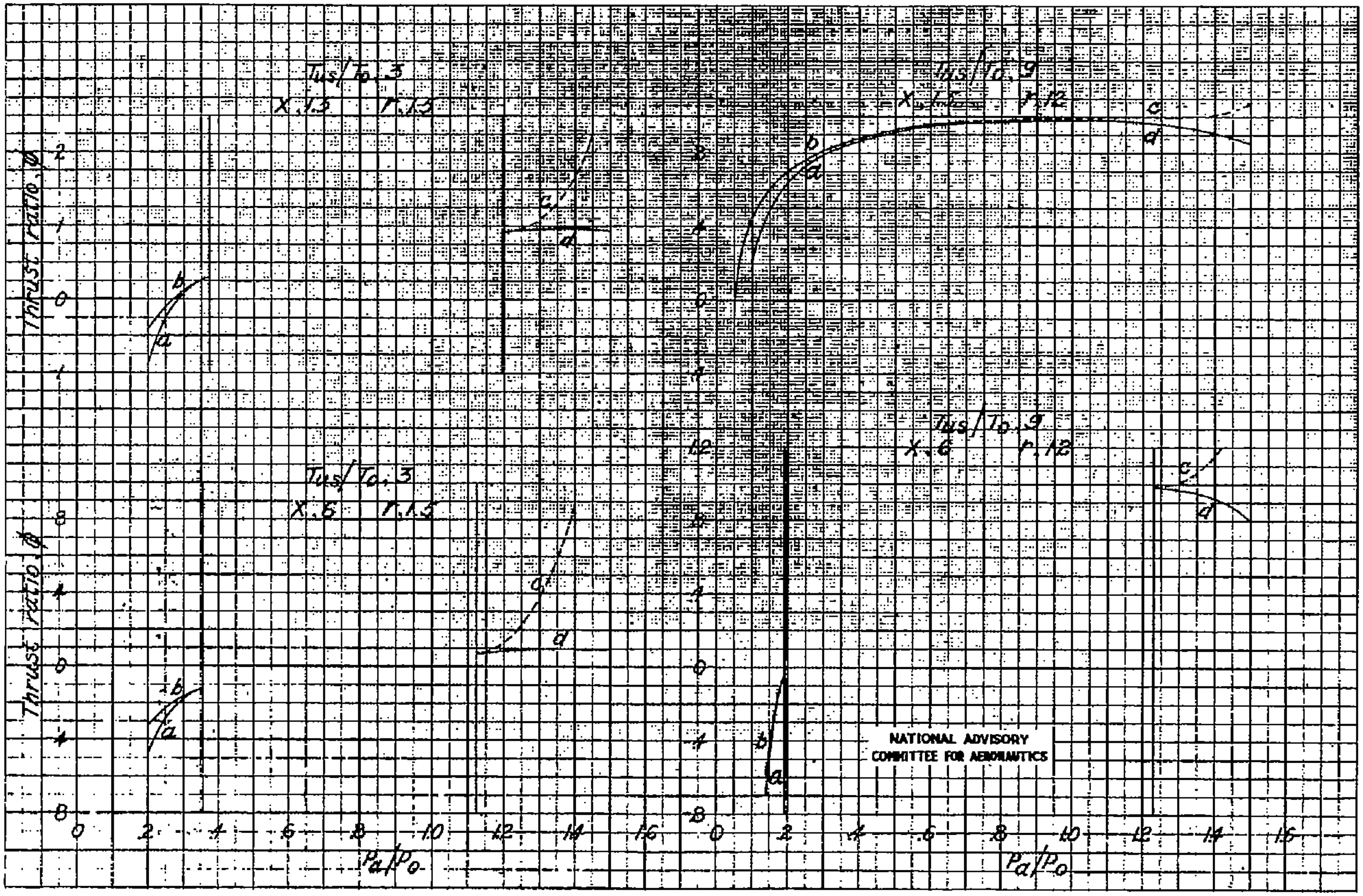


Figure 15.- Thrust augmentation of turbojet installations with a nozzle at ejector mixing-tube exit (M_0, 0.8).

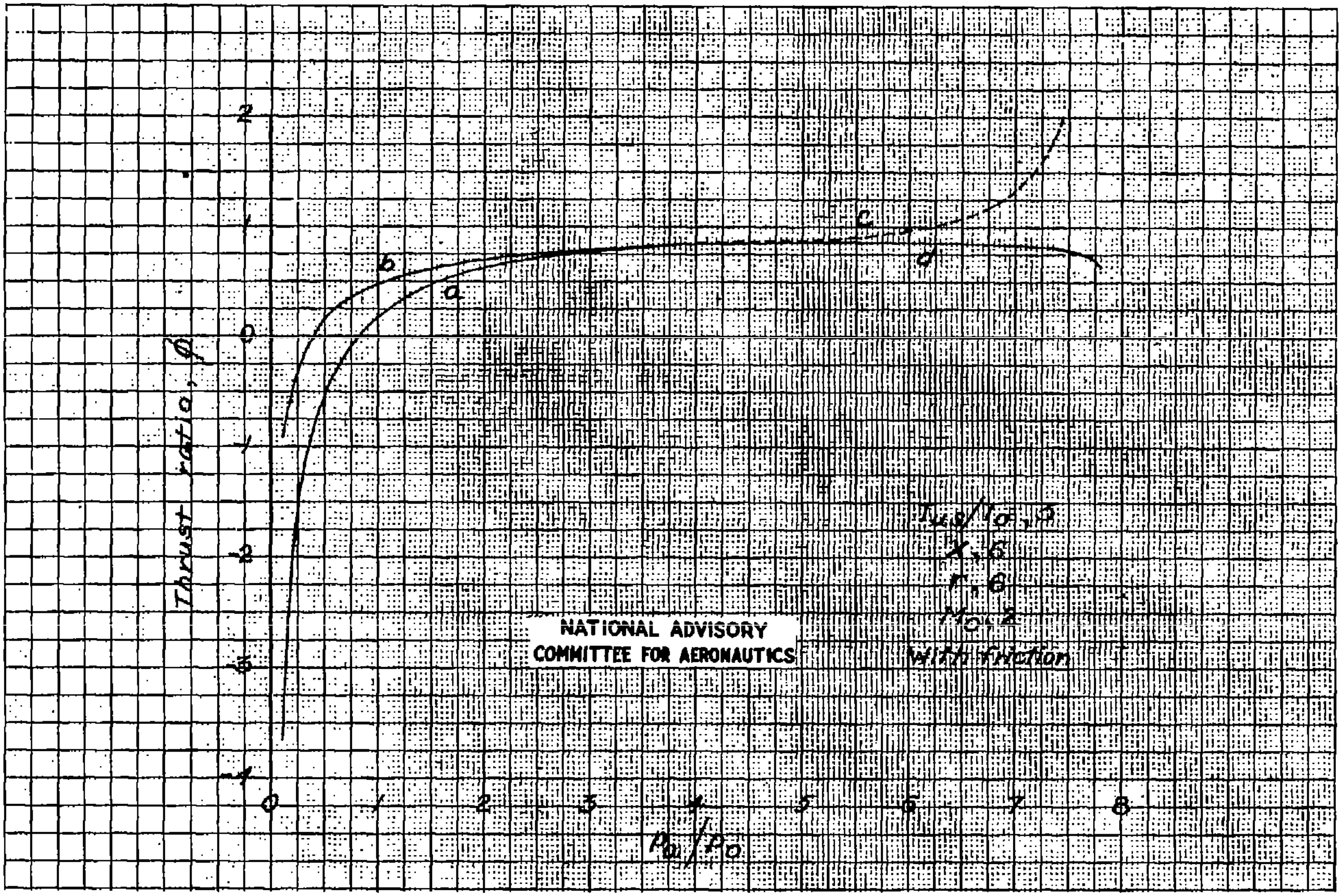


Figure 16.- Thrust augmentation of turbojet installations with a nozzle at ejector mixing-tube exit (M_0 , 2.0).

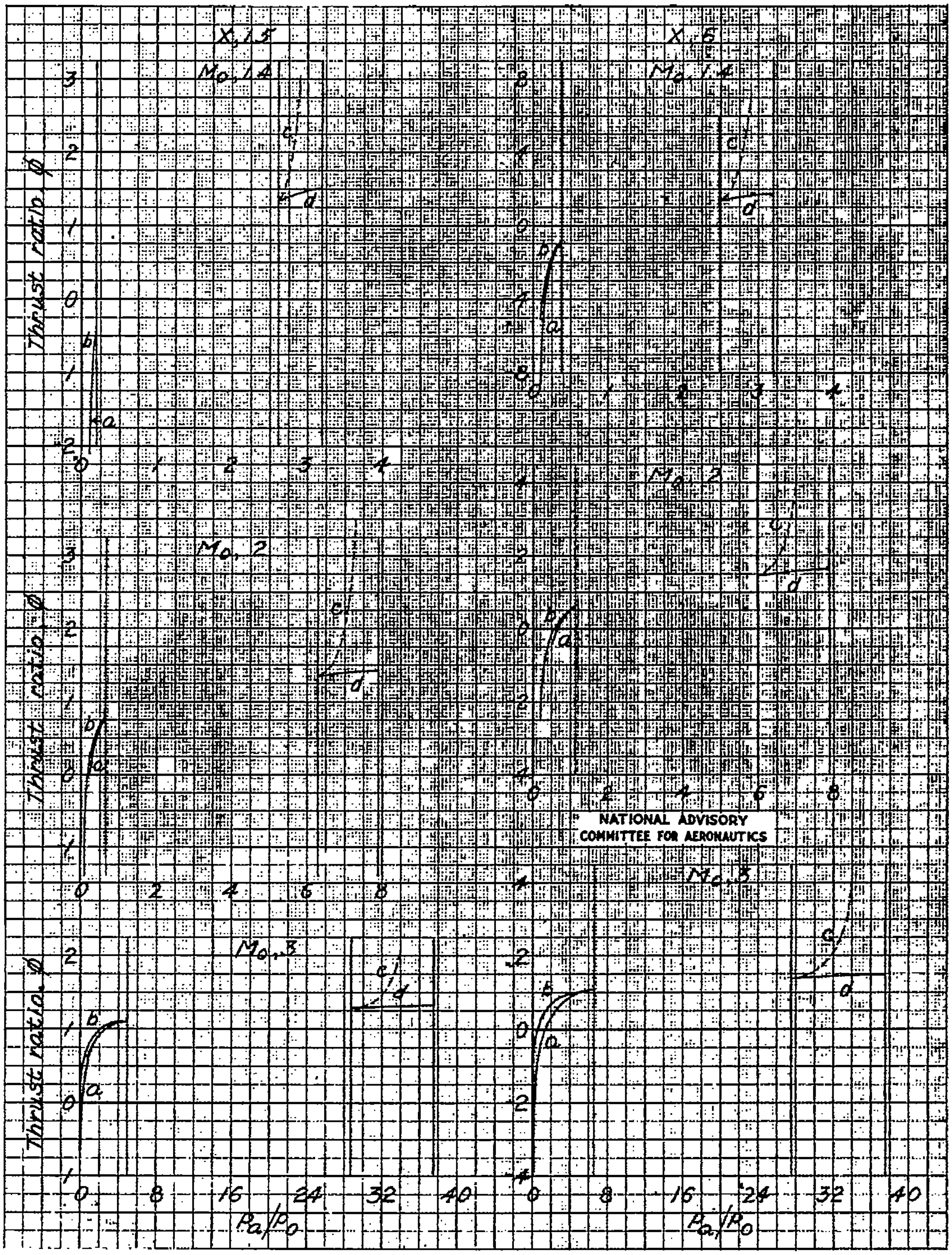


Figure 17.- Thrust augmentation of ram-jet installations with a nozzle at ejector mixing-tube exit (r, 1.0; air-fuel ratio, 15).